Imperfect Banking Competition and Macroeconomic Volatility: a DSGE Framework

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Abstract

Following the recent financial crisis, there has been an increasing focus on incorporating financial frictions into a dynamic stochastic general equilibrium (DSGE) model, often by introducing the agency problem which serves to amplify macroeconomic shocks. This paper examines the impact of another important financial friction, imperfect competition in banking, on aggregate fluctuations by incorporating a Cournot banking sector into a DSGE model embedded with the agency problem that gives rise to collateral constraints. In the presence of a binding collateral constraint, imperfect banking competition is found to have an amplification effect on aggregate fluctuations after a contractionary monetary policy shock and adverse collateral shocks. Adverse shocks that make borrowers more financially constrained and their loan demand more inelastic can induce banks with market power to raise the loan rate, resulting in a countercyclical loan interest margin that amplifies the aggregate fluctuations.

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1 Introduction

The recent financial crisis has shown that incorporating financial frictions into a dynamic stochastic general equilibrium (DSGE) model is important as they tend to be a critical propagation mechanism of macroeconomic shocks. So far, the most commonly used financial friction is the agency problem, which is often introduced via a monitoring cost that gives rise to the external finance premium[1] (Bernanke, Gertler and Gilchrist, 1999) or costly debt enforcement in the framework of collateral constraints (Kiyotaki and Moore, 1997). As agents' balance sheet conditions worsen during bad times, indicating more severe agency problems, the resulting increase in the external finance premium (EFP) or the tightness of the collateral constraint tends to amplify the initial shock that adversely affects the balance sheet conditions.

This paper focuses on another important financial friction, imperfect competition in banking, that can also affect macroeconomic fluctuations via an endogenously changing real loan interest margin. In this paper, bank deposits denominated in nominal terms and risk-free nominal bonds are assumed to be perfect substitutes under full deposit insurance, so the nominal deposit rate is equal to the nominal interest rate on risk-free nominal bonds which is assumed to be controlled by the central bank. The real loan interest margin in this paper refers to the wedge between the real loan rate and the real deposit rate.

Imperfect banking competition is modelled by a Cournot banking sector and is incorporated into a model embedded with the agency problem arising from costly debt enforcement in the framework of collateral constraints (Kiyotaki and Moore, 1997). The collateral constraint is tied to the value of housing and the value of physical capital. Within the model, two types of banking competition, perfect and imperfect banking competition, are introduced one at a time. By introducing imperfect banking competition into the framework of the agency problem, the potential interaction effect between the two financial frictions can be found. It is found in this paper that when imperfect banking competition is acting together with the agency problem, it tends to amplify the responses of output, physical capital and investment after a contractionary monetary shock and negative collateral shocks, but slightly attenuate the responses of those variables after a negative productivity shock.[2]

The amplification effect of imperfect banking competition after the contractionary monetary shock and the negative collateral shocks can be explained by the countercyclical real loan margin which is caused by a joint effect between banks' market power and time-varying price elasticities of loan demand facing the banks. In this model, the changing degree of

[1]The external finance premium is the additional cost paid by borrowers to compensate for risks when tapping external funding sources.

[2]In this paper, collateral shocks refer to the shocks to the loan-to-value (LTV) ratios.

tightness of the binding borrowing constraint is one of the main driving forces behind the changing loan demand elasticities over the business cycle. During bad times, the borrowing constraint is more tightly binding and the borrowers are more financially constrained, making the loan demand more inelastic. When the loan demand becomes more inelastic after a negative shock,[3] banks with market power under Cournot competition can take advantage of this reduction in elasticity to maximize profits, leading to a higher loan rate in a Cournot equilibrium. For a given deposit rate set by the central bank, the higher loan rate means that the loan margin rises after the negative shock.

However, in spite of a countercyclical real loan margin after the negative productivity shock, there is a slight attenuation effect under imperfect banking competition. This can be explained by two possible reasons. First, the magnitude of the countercyclical real loan margin after the negative productivity shock is much smaller compared to all the other cases, due to a smaller decrease in the loan demand elasticity on impact. Second, a net worth effect arising from the interaction between the agency problem and imperfect banking competition appears to have an attenuation effect on aggregate fluctuations after the negative productivity shock. As a result, it appears that the attenuation effect from the net worth channel dominates the amplification effect from the countercyclical real loan margin after the negative productivity shock.

This paper contributes to the existing literature in two ways. First, imperfect banking competition is modelled by a Cournot banking sector. Current literature often uses the Dixit and Stiglitz (1977) model of monopolistic competition to analyse imperfect banking competition in DSGE models (Gerali et al., 2010; Hülsewig, Mayer and Wollmershäuser, 2009), which requires unrealistic assumptions on agents' preferences for banking. In reality, people tend to rely mostly on one bank and do not demand a composite bundle of many loan or deposit contracts. Second, this paper derives an explicit expression for the price elasticity of loan demand and formally analyses how the loan demand elasticity changes in response to different shocks.

The remainder of the paper is structured as follows. Section 2 summarizes the existing literature on empirical evidence for imperfect banking competition and DSGE modelling with financial frictions. Section 3 introduces the model to analyse the effect of imperfect banking competition in a DSGE model embedded with the agency problem. Section 4 explains the choice of parameter calibration. Section 5 shows the impulse responses of some key variables to a contractionary monetary shock, a negative productivity shock and negative collateral shocks. In each case, the impulse responses of a given variable under the two types of banking

[3] In this paper, a negative shock refers to a contractionary monetary shock, a negative productivity shock and negative collateral shocks.

competition are compared and discussed. Section 6 concludes.

2 Literature Review

Empirical evidence on imperfect banking competition is summarized in Section 2.1. Three common approaches of modelling the agency problem in a DSGE model are shown in Section 2.2.1. The motivation for using imperfect banking competition to generate the time-varying spread is discussed in Section 2.2.2. Finally, Section 2.2.3 summarizes the literature on DSGE modelling incorporating both the agency problem and imperfect banking competition.

2.1 Empirical Evidence on Imperfect Banking Competition

There is a wide range of measures used to examine the competition level of the banking industry empirically. Nevertheless, a common finding is that banks indeed have market power and competition levels vary across countries and over time (De Bandt and Davis, 2000). Bikker and Haaf (2002) examined the degree of competition in European banking industries using the Panzar and Rosse (1987) method and found evidence for monopolistic competition in most countries. Ehrmann et al. (2001) found a non-negligible degree of imperfect banking competition in the Euro area. Looking at specific countries, Oxenstierna (1999) analysed the Swedish bank oligopoly empirically and found significant market power in both the loan and deposit market. In addition, Berg and Kim (1998) found evidence for significant oligopolistic behavior (or strategic interactions) of banks competing in the retail banking sector using a three-year panel data from the Norwegian banking sector. A Cournot banking sector is used in this paper to model imperfect banking competition to characterize the oligopolistic nature of the banking sector.

2.2 Financial Frictions in DSGE Models

2.2.1 Agency Problem

The agency problem between borrowers and lenders is often introduced in DSGE models to generate the *financial accelerator* effect (Bernanke, Gertler and Gilchrist, 1996). As noted by Bernanke, Gertler and Gilchrist (1999), a worsening of information asymmetries between borrowers and lenders and the associated increase in agency costs characterize most of the potential problems in financial markets. In general, there are three commonly used methods of incorporating the agency problem into a DSGE model, aiming to analyse a variety of different issues. First, the agency problem can be introduced via costly debt enforcement and collateral constraints, following the tradition of Kiyotaki and Moore (1997). In their original model set-up, it is assumed that only farmers have the technology to increase the value of land by growing fruit but they cannot precommit to work. Hence, there is a possibility that farmers would want to threaten the creditors by repudiating their debt contracts. Given the possibility of debt repudiation, creditors would collateralize the farmer's land and only grant a loan with a value that does not exceed the value of the collateral. In general, the agency problem in this context is that borrowers cannot be forced to repay unsecured debt (Beck, Colciago and Pfajfar, 2014). Iacoviello (2005) tied the collateral constraints to borrowers' housing values and found a non-uniform financial accelerator effect due to the nominal debt, of which the resulting debt deflation effect amplifies a housing price shock but attenuates an inflation shock.

Second, the agency problem can be modelled by the costly state verification of Townsend (1979) and assuming that the idiosyncratic return on a project is private information to the borrowers and cannot be costlessly observed by the lenders. Due to the ex post information asymmetry, borrowers have an incentive to default by untruthfully declaring a low return even when they have the ability to repay, in which case the lenders need to incur a monitoring cost to verify the return. The optimal contract then involves a trade-off between the monitoring cost and the default probability, leading to an endogenous external finance premium (EFP) that raises the cost of borrowing and amplifies business cycle fluctuations (Agénor and Montiel, 2015). Using this approach, Carlstrom and Fuerst (1997) found the agency cost can generate hump-shaped output behavior after a productivity shock due to a hump-shaped response of the entrepreneurial net worth. Bernanke, Gertler and Gilchrist (1999) found that in the presence of a countercyclical EFP,[4] the impact response of output to a given monetary shock is around 50% greater than that of the frictionless case. Using an estimated DSGE model based on the US data over the period of 1973-2009, Gilchrist, Ortiz and Zakrajsek (2009) found an increase in the EFP can cause a significant and prolonged decline in output and investment.

Third, more recent literature on DSGE models with the agency problem (Gertler and Karadi, 2011; Gertler, Kiyotaki et al., 2010) tends to focus on financial frictions at the level of the financial intermediaries in order to analyse unconventional monetary policy, i.e., financial intermediaries themselves may face difficulty in obtaining funds from creditors. The agency problem is introduced by assuming that bankers can divert a fraction of assets for their own personal gain and default on their liabilities. Since the creditors are aware of this default possibility, they limit the amount of lending to bankers. In fact, creditors are only willing to lend to bankers subject to a binding incentive constraint for the bankers. Gertler,

[4] Countercyclical EFP means EFP rises during bad times.

Kiyotaki and Queralto (2012) used this approach to quantitatively analyze the impact of direct central bank lending and found anticipated government credit policy would induce banks to adopt a more risky balance sheet.[5] Gertler and Karadi (2011) found that credit policy can significantly moderate the decline in investment by dampening the rise in the spread.[6]

However, in all these papers where the financial intermediation is explicitly modelled, the banking sector is assumed to be perfectly competitive, which is at odds with the empirical evidence discussed in Section 2.1. Instead of assuming that perfectly competitive banks simply accommodate any changes in the demand side of the credit market arising from the variation in the severity of the agency problem, Gerali et al. (2010) recognized that imperfect competition is a key supply-side feature of the banking sector that should not be neglected.

2.2.2 Imperfect Banking Competition

Imperfect banking competition can potentially help explain business cycle fluctuations via a varying interest rate spread. The role of a time-varying spread in understanding business cycle fluctuations has been explored in the literature (Cúrdia and Woodford, 2015; Gertler and Karadi, 2011; Gertler, Kiyotaki et al., 2010; Gilchrist, Ortiz and Zakrajsek, 2009), although a variety of different assumptions, other than imperfect banking competition, have been used to generate the spread. Gilchrist, Ortiz and Zakrajsek (2009) used the corporate credit spread to proxy for the EFP, which is generated by the agency problem, and found an increasing EFP can cause significant and persistent falls in output and investment. Cúrdia and Woodford (2015) introduced a time-varying spread by assuming that the loan-origination process would consume real resources and that there is an exogenously varying loss rate on loans. They found that augmenting the standard Taylor rule with an additional credit spread term can reduce the distortion from a disturbance that raises the equilibrium spread. Similarly, Goodfriend and McCallum (2007) assumed bank loans and deposits are produced by a competitive banking sector according to a Cobb-Douglas production function. The costly production process gives rise to an EFP, which can be procyclical or countercyclical in response to a monetary shock depending on different parameter calibration.

There are a few attempts to incorporate imperfect banking competition into a DSGE model to generate the time-varying spread. For example, monopolistic competition within the Dixit and Stiglitz (1977) framework is often used to model imperfect banking competition

[5] Credit policy refers to the government or central bank intervention by directly injecting credit in response to changes in the credit spread, according to a feedback rule.

[6] The spread is the difference between the loan rate and the deposit rate, which is equivalent to the loan interest margin in this paper. In a model with EFP and perfect banking competition, the spread often refers to the difference between the expected return on capital and the risk-free interest rate.

(Gerali et al., 2010; Hülsewig, Mayer and Wollmershäuser, 2009). However, assuming agents demand a composite CES basket of loan and deposit contracts is unrealistic, given that in reality, people do not have a 'taste for variety' preference for banking as they often rely on only one bank. Andrés and Arce (2012) avoided this problem by using a dynamic version of Salop's (1979) model of imperfect competition. In Salop's (1979) circular city model, borrowers are uniformly distributed along the circumference of a unit circle and each of them only borrows from one bank. By contrast, imperfect banking competition is modelled by a Cournot banking sector in this paper and the time-varying loan margin or the spread is generated by a combination of banks' market power and time-varying elasticities of loan demand facing each bank under Cournot competition.

2.2.3 Interaction Between Agency Problem and Imperfect Bank Competition

Hafstead and Smith (2012) used the monopolistic competition of Dixit and Stiglitz (1977) to model imperfect banking competition, which is incorporated into the financial accelerator framework of Bernanke, Gertler and Gilchrist (1999). They found that with both the agency problem and monopolistic competition in the banking sector, the responses of aggregate variables to a positive productivity shock tend to be reduced relative to a DSGE model without the financial accelerator. Furthermore, monopolistic competition in banking is found to mitigate the impact of the agency problem. Using the same modelling approach for the two financial frictions in an estimated DSGE model, Dib (2010) found the banking sector tends to dampen the fluctuations after a monetary shock. Monopolistic competition à *la* Dixit and Stiglitz (1977) for the banking sector is also used in Gerali et al. (2010), while the agency problem is introduced via costly debt enforcement. The inclusion of banking is found to attenuate the impact of a contractionary monetary shock mainly due to the special feature of quadratic adjustment costs in changing the interest rates in their paper. Andrés and Arce (2012) incorporated the Salop (1979) model of monopolistic competition for the banking sector into the framework of collateral constraints and found the negative response of output to a contractionary monetary shock is larger and more persistent under stronger banking competition.

In this paper, the agency problem in the extended model is introduced using the approach of Kiyotaki and Moore (1997), because as will be shown in Section 3, the binding collateral constraint provides a convenient way of modelling the market loan demand, which makes it easier to introduce imperfect banking competition. Although Hafstead and Smith (2012) and Dib (2010) incorporated imperfect banking competition into the framework of EFP, they used the monopolistic competition of Dixit and Stiglitz (1977) to model the banking competition. Under monopolistic competition, each individual bank sets its own interest rates, taking the market interest rate and the market loan demand and deposit supply as given. This assumption is convenient in solving the banks' problem. By contrast, modelling the banking sector as a Cournot oligopoly requires working with the market loan demand directly and this makes it difficult to solve in the framework of Bernanke, Gertler and Gilchrist (1999).[7]

This paper closely follows Andrés and Arce (2012) by modelling both imperfect banking competition and the agency problem and using the binding borrowing constraint to solve the bank's problem. However, there are three important modelling differences from their paper. First, the banking sector is modelled as a Cournot oligopoly, which leads to a major discrepancy from their paper in terms of solving banks' optimisation problem. Second, two collateral assets, physical capital and housing, are introduced in this paper, while housing is used as the only collateral asset in their paper. As will be shown in Section 6, these two modelling choices can produce a countercyclical real loan margin that is large enough to amplify the response of output. Despite the real loan margin being countercyclical in Andrés and Arce (2012), the response of which is quite small, the response of output is attenuated under imperfect banking competition after a contractionary monetary policy shock in their paper. Third, instead of assuming the entrepreneurs (or borrowers) rent physical capital from the households in every period, it is assumed that the entrepreneurs can invest in physical capital in this paper. In this way, two types of collateral assets can be introduced in the model to generate the amplification effect under Cournot banking competition.

The findings in this paper are new to the literature, in the sense that imperfect banking competition can amplify aggregate fluctuations after a contractionary monetary shock due to a countercyclical real loan margin. Although Andrés and Arce (2012) also found a countercyclical loan margin after a contractionary monetary shock, its response is too small to amplify the response of output.

3 The Model

The model aims to show the effect of imperfect banking competition relative to perfect banking competition on aggregate fluctuations in a framework of collateral constraints. To introduce the financial intermediary into a DSGE model, savings and investment decisions cannot be made by the same representative agent, as in a standard DSGE model without the banking sector, where the representative household owns physical capital and invests in it directly. In this model, households supply funds to the banking sector via bank deposits, while investment decisions are made by entrepreneurs who borrow from the banking sector

[7]Since the threshold below which the borrowers would choose to default also depends on the market loan rate, it is difficult to solve the banks' problem in this context.

to purchase new physical capital and real estate. The model set-up for perfect banking competition in a framework of collateral constraints is described in Section 3.1 before introducing imperfect banking competition in Section 3.2.

3.1 Perfect Banking Competition Benchmark

There are six types of agents: households, entrepreneurs, retailers, capital producers, banks, and a central bank. Each of the former five agent types has a unit mass. There is a fixed housing supply that can be invested by households and entrepreneurs, following Iacoviello (2005) and Andrés and Arce (2012). Households consume, supply labor to the entrepreneurs, invest in housing and decide how much to save via one-period non-state-contingent nominal bank deposit contracts or one-period risk-free nominal bonds. Perfectly competitive entrepreneurs are born with some physical capital and housing in the initial period and they have access to a Cobb-Douglas production technology. They hire labor from households, purchase new capital from capital producers and purchase real estate from the households to produce a wholesale good. The wholesale good produced by entrepreneurs cannot be consumed directly and is sold to monopolistically competitive retailers who then differentiate the wholesale good costlessly into different varieties. Each retailer uses the wholesale good as the only input to produce a different variety. The final consumption good is a composite CES (constant elasticity of substitution) bundle of all the varieties. Perfectly competitive capital producers buy the undepreciated capital from entrepreneurs and consumption goods from retailers to produce new capital, which is then sold back to the entrepreneurs.

Banks offer two types of one-period contracts: deposit contracts and loan contracts. The contracts are denominated in nominal terms, which means they are not inflation-indexed and the borrowing or saving decisions are made on the basis of a preset contractual nominal loan or deposit rate. Assuming nominal bank deposits and one-period riskless nominal bonds are perfect substitutes to households under full deposit insurance, the gross nominal deposit rate must equal the gross nominal interest rate R_t earned on the riskless nominal bond invested in period t. Since banks are perfectly competitive, each of them takes the nominal loan rate as given and maximizes its profit with respect to the loan (or deposit) quantity. Assuming costless financial intermediation and no default, the gross nominal loan rate $R_{b,t}$ equals the gross nominal risk-free interest rate R_t which is controlled by the central bank.

3.1.1 Households

There is a continuum of unit mass of identical infinitely-lived households. The representative household maximizes the following expected utility:

$$E_t \sum_{s=0}^{\infty} \beta^s [ln(c_{t+s}) + \phi ln(1 - l_{t+s}) + \phi_h ln(h_{t+s})] \tag{1}$$

which depends on consumption c, labor supply l and real estate holdings h, with E_t being the expectation operator conditional on information in period t, $\beta \in (0, 1)$ being the subjective discount factor of the household, $\phi > 0$ and $\phi_h > 0$. The total time endowment is normalised to 1, so $(1 - l_t)$ denotes the amount of period-t leisure time. ϕ and ϕ_h are the relative utility weights on leisure time and housing respectively. As in Gertler and Karadi (2011), a cashless economy is considered here for the convenience of neglecting the real money balances in the utility function.

In each period t, the household consumes c_t, saves d_t units of one-period bank deposits[8] in real (final consumption) terms, invests in housing h_t and supplies labor hours l_t. The nominal deposits saved in period $t-1$ earn a gross nominal interest rate R_{t-1} at the beginning of period t. Let p_t denote the unit price of the final consumption good, then the gross inflation rate is $\pi_t \equiv \frac{p_t}{p_{t-1}}$. Assume retailers, capital producers and banks are owned by the households. Given the gross real interest earnings on deposits $\frac{R_{t-1}d_{t-1}}{\pi_t}$ at the beginning of period t, real labor income $w_t l_t$ and real lump-sum profits Π_t^R, Π_t^{CP} and Π_t^B made by retailers, capital producers and the banking sector respectively, the household decides how much to consume and save and how much housing investment $(h_t - h_{t-1})$ to make in period t. With a perfectly competitive banking sector, the expected profit is zero, however, the profit can differ from zero ex post. Assuming there is no depreciation of housing, the representative household faces the following budget constraint:

$$c_t + d_t + q_{h,t}(h_t - h_{t-1}) = \frac{R_{t-1}d_{t-1}}{\pi_t} + w_t l_t + \Pi_t^R + \Pi_t^{CP} + \Pi_t^B \tag{2}$$

where $q_{h,t}$ is the real price of housing. Let λ_t denote the Lagrange multiplier associated with the budget constraint or equivalently, the marginal utility of consumption. The first order conditions with respect to consumption c_t (3), labor supply l_t (4), housing h_t (5), and bank deposits d_t (6) are as follows:

$$\lambda_t = \frac{1}{c_t} \tag{3}$$

[8] Assume there is zero net supply of risk-free nominal bonds, so in equilibrium, households hold only nominal bank deposits.

$$\frac{\phi}{1-l_t} = \lambda_t w_t \tag{4}$$

$$\frac{\phi_h}{h_t} + \beta E_t(\lambda_{t+1} q_{h,t+1}) = \lambda_t q_{h,t} \tag{5}$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \tag{6}$$

Equation (6) is the standard intertemporal Euler equation, which can also be written as:

$$1 = E_t \left[\Lambda_{t,t+1} \frac{R_t}{\pi_{t+1}} \right] \tag{7}$$

where $\Lambda_{t,t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the stochastic discount factor in period t for real payoffs in period $t+1$.

3.1.2 Entrepreneurs

In period $t-1$, a continuum of mass 1 of perfectly competitive entrepreneurs acquire physical capital k_{t-1} from capital producers at the real price q_{t-1} and real estate h^E_{t-1} from households at the real price $q_{h,t-1}$ for production in period t. k_{t-1}, h^E_{t-1} and labor l^E_t hired from households are used as inputs to produce the wholesale good in period t using a constant-returns-to-scale Cobb-Douglas production technology:

$$y_{w,t} = z_t k^{\alpha}_{t-1} (h^E_{t-1})^v (l^E_t)^{1-\alpha-v} \tag{8}$$

where $\alpha \in (0,1)$ and $v \in (0,1)$ are the output elasticities of physical capital and housing respectively. $y_{w,t}$ is the output of the wholesale good (which differs from the output of the final consumption good by a factor of the price dispersion as will be shown by equation (36) in Section 3.1.4). z_t is the productivity shock that follows an AR(1) process in logs:

$$lnz_t = \psi lnz_{t-1} + e_{z,t} \tag{9}$$

with $\psi \in (0,1)$ indicating the persistence of the process, and $e_{z,t}$ normally distributed with mean zero and standard deviation σ_z.

Let β^E denote the subjective discount factor for entrepreneurs. Following Iacoviello (2005), it is assumed that $\beta^E < \beta$ to ensure that in the steady state and its neighborhood, entrepreneurs are net borrowers and households are net savers. The necessity of this assumption is shown later after solving the entrepreneur's problem. The entrepreneur's objective is to maximize the expected lifetime utility:

$$E_t \sum_{s=0}^{\infty} (\beta^E)^s ln(c_{t+s}^E) \tag{10}$$

subject to a budget constraint (11) and a collateral constraint (12). Let $R_{b,t}$ denote the gross nominal loan rate in period t, then at the beginning of period $t+1$, the gross real loan interest payment is $\frac{R_{b,t}b_t}{\pi_{t+1}}$. Since the loan contract is denominated in nominal terms with a specified $R_{b,t}$, a rise in inflation in period $t+1$ reduces the firm's real debt burden. At the end of period t, entrepreneurs can sell the undepreciated capital $(1-\delta)k_{t-1}$ to capital producers at the real price of capital q_t, where $\delta > 0$ is the depreciation rate for physical capital. The wholesale good produced in period t is sold to retailers at a nominal price $p_{w,t}$. Let x_t denote the markup of the price of the final consumption good over the price of the wholesale good, that is, $x_t \equiv \frac{p_t}{p_{w,t}}$. In each period t, the outflow of funds due to consumption c_t^E, cost of capital investment $q_t[k_t - (1-\delta)k_{t-1}]$, cost of housing investment $q_{h,t}(h_t^E - h_{t-1}^E)$, real wage payments to households $w_t l_t^E$ and real gross loan interest payment $\frac{R_{b,t-1}b_{t-1}}{\pi_t}$, would equal the inflow of funds due to the real revenue from selling the wholesale good $\frac{y_{w,t}}{x_t}$ and the loans granted by banks b_t. Hence the budget constraint in real terms is:

$$c_t^E + q_t k_t + q_{h,t} h_t^E + w_t l_t^E + \frac{R_{b,t-1}b_{t-1}}{\pi_t} = \frac{y_{w,t}}{x_t} + (1-\delta)q_t k_{t-1} + q_{h,t} h_{t-1}^E + b_t \tag{11}$$

The agency problem in the framework of collateral constraints is often introduced by assuming costly debt enforcement, based on Kiyotaki and Moore (1997). Assume the entrepreneurs face limited debt obligations and if they repudiate their debt obligations, banks can only claim a fraction of their assets. Consequently, the maximum amount an entrepreneur can borrow is equivalent to the maximum amount of assets that banks can claim after the repudiation of debt. Assume both real estate and physical capital can be used as collateral assets,[9] then the collateral constraint can be written as:

$$b_t \leqslant m_{h,t} E_t \left[\frac{q_{h,t+1} h_t^E \pi_{t+1}}{R_{b,t}} \right] + m_{k,t} E_t \left[\frac{q_{t+1} k_t (1-\delta) \pi_{t+1}}{R_{b,t}} \right] \tag{12}$$

where $m_h \in (0,1)$ and $m_k \in (0,1)$ reflect the fractions of the values of housing collateral and physical capital collateral that can be recouped by banks when the entrepreneurs fail to repay the debt respectively. $m_{h,t}$ and $m_{k,t}$ are the collateral shocks that follow an AR(1) process in logs:

$$lnm_{h,t} = \psi_{m_h} lnm_{h,t-1} + e_{m_h,t} \tag{13}$$

[9] From this section onwards, assets refer to both real estate and physical capital.

$$lnm_{k,t} = \psi_{m_k} lnm_{k,t-1} + e_{m_k,t} \tag{14}$$

with $\psi_{m_h} \in (0,1)$ and $\psi_{m_k} \in (0,1)$ indicating the persistence of the process, $e_{m_h,t}$ and $e_{m_h,t}$ normally distributed with mean zero and standard deviations σ_{m_h} and σ_{m_k} respectively.

As can be seen from (12), the maximum amount of borrowing in real terms b_t is bounded by $m_{h,t}E_t\left[\frac{q_{h,t+1}h_t^E\pi_{t+1}}{R_{b,t}}\right] + m_{k,t}E_t\left[\frac{q_{t+1}k_t(1-\delta)\pi_{t+1}}{R_{b,t}}\right]$, which is also the amount of assets that banks can claim after the debt repudiation. Although real estate is the only collateral in both Iacoviello (2005) and Andrés and Arce (2012), it is plausible to assume physical capital can also serve the purpose.

Let $\lambda_{1,t}^E$ and $\lambda_{2,t}^E$ denote the Lagrange multipliers associated with the budget constraint (11) and the borrowing constraint (12) respectively, the first order conditions with respect to the entrepreneur's consumption c_t^E (15), loan demand b_t (16), labor demand l_t^E (17), capital demand k_t (18) and housing demand h_t^E (19) are:

$$\frac{1}{c_t^E} = \lambda_{1,t}^E \tag{15}$$

$$\lambda_{2,t}^E = \lambda_{1,t}^E - \beta^E E_t\left(\lambda_{1,t+1}^E \frac{R_{b,t}}{\pi_{t+1}}\right) \tag{16}$$

$$\frac{(1-\alpha-v)y_{w,t}}{x_t l_t^E} = w_t \tag{17}$$

$$q_t\lambda_{1,t}^E = \beta^E E_t\left\{\lambda_{1,t+1}^E\left[\frac{\alpha y_{w,t+1}}{x_{t+1}k_t} + (1-\delta)q_{t+1}\right]\right\} + \lambda_{2,t}^E m_{k,t}E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}}\right] \tag{18}$$

$$q_{h,t}\lambda_{1,t}^E = \beta^E E_t\left\{\lambda_{1,t+1}^E\left[\frac{v y_{w,t+1}}{x_{t+1}h_t^E} + q_{h,t+1}\right]\right\} + \lambda_{2,t}^E m_{h,t}E_t\left[\frac{q_{h,t+1}\pi_{t+1}}{R_{b,t}}\right] \tag{19}$$

Let variables without the time subscript denote the steady state values. Combining (15) and (16), it can be seen that in the steady state:

$$\lambda_2^E = \frac{1}{c^E}\left(1 - \beta^E\frac{R_b}{\pi}\right) \tag{20}$$

From Euler equation (7) derived from the household's problem in Section 3.1.1, the steady state value of the gross real interest rate $\frac{R}{\pi}$ is determined by the household's subjective discount factor, such that $\frac{R}{\pi} = \frac{1}{\beta}$. Since $R_{bt} = R_t$ under perfect banking competition, $\lambda_2^E = \frac{1}{c^E}\left(1 - \frac{\beta^E}{\beta}\right)$. To ensure that the borrowing constraint always binds in the steady state, λ_2^E must be positive, which implies $\beta^E < \beta$. This heterogeneity in the subjective discount factors guarantees that in the steady state, impatient entrepreneurs are net borrowers. In this strand of literature, it is a common approach to assume $\beta^E < \beta$ to ensure the borrowing constraint always binds in the steady state and its neighborhood, as long as the size of the shocks are sufficiently small (Liu, Wang and Zha, 2013; Andrés and Arce, 2012; Gerali et al., 2010; Iacoviello, 2005). As Liu, Wang and Zha (2013) pointed out, it is difficult to capture the potential nonlinearity arising from an occasionally binding borrowing constraint when the model is solved on the basis of a log-linearized equilibrium system. Nevertheless, due to a large number of endogenous variables, they also follow the common approach in the literature by assuming the borrowing constraint is always binding. To ensure the borrowing constraint is always binding in this paper, not only is the parameter restriction imposed to guarantee a positive λ_2^E, but also only negative shocks are analysed and the impulse response of $\lambda_{2,t}^E$ is also shown in Section 5.

Based on the budget constraint (11), define the entrepreneur's net worth nw_t as the share of revenue accruing to the factor inputs of physical capital and real estate $\frac{(\alpha+v)y_{w,t}}{x_t}$, plus the total value of the real estate holdings and capital stock $q_{h,t}h_{t-1}^E + q_t(1-\delta)k_{t-1}$, and net of the gross real loan interest payment $\frac{R_{b,t-1}b_{t-1}}{\pi_t}$ at the beginning of period t. Hence, nw_t can be written as:

$$nw_t \equiv \frac{(\alpha+v)y_{w,t}}{x_t} + q_t(1-\delta)k_{t-1} + q_{h,t}h_{t-1}^E - \frac{R_{b,t-1}b_{t-1}}{\pi_t} \tag{21}$$

where $\frac{(\alpha+v)y_{w,t}}{x_t} = \frac{y_{w,t}}{x_t} - w_t l_t^E$, which can be proved using the first order condition with respect to l_t^E (17). Rewriting the budget constraint (11) in terms nw_t would give:

$$c_t^E + q_t k_t + q_{h,t}h_t^E = nw_t + b_t \tag{22}$$

which implies the entrepreneur finances his consumption c_t^E and the purchase of new capital and housing $(q_t k_t + q_{h,t}h_t^E)$ by bank loans b_t and the retained earnings nw_t. Using the first order conditions (15)-(19) and the borrowing constraint (12) that is binding, it is proved in Appendix B that due to the assumption of log utility, the entrepreneur's consumption in period t is a fixed proportion $(1-\beta^E)$ of nw_t, that is:

$$c_t^E = (1-\beta^E)nw_t \tag{23}$$

The real loan demand b_t is the total purchasing cost of new capital and housing in excess of the internal financing or the savings $\beta^E nw_t$:

$$b_t = q_t k_t + q_{h,t}h_t^E - \beta^E nw_t \tag{24}$$

The derivation is shown in Appendix B. In each period t, the entrepreneur consumes a fraction of the net worth nw_t (or retained earnings), and saves the rest of nw_t to partly finance the purchase of physical capital and real estate, assuming entrepreneurs are born with some physical capital and housing. In the presence of the collateral constraint, borrowers can only access a limited amount of bank credit and hence they would want to accumulate retained earnings over time to be used as internal financing and become less financially constrained.

3.1.3 Capital Producers

Perfectly competitive capital producers are introduced to derive an explicit expression for the real price of capital q_t (Gambacorta and Signoretti, 2014).[10] They purchase undepreciated capital $(1-\delta)k_{t-1}$ at the real price q_t from entrepreneurs and i_t units of final consumption goods from retailers to produce new capital k_t at the end of period t: $k_t = i_t + (1-\delta)k_{t-1}$, where i_t is also net investment. The new capital produced will be sold back to the entrepreneur at the real price q_t, which will be used in the production of the wholesale good in period $t+1$. Following Christiano, Eichenbaum and Evans (2005), assume capital producers face investment adjustment costs that depend on the gross growth rate of investment $\frac{i_t}{i_{t-1}}$. Assume old capital can be converted one-to-one into new capital and quadratic unit investment adjustment cost $f\left(\frac{i_t}{i_{t-1}}\right) = \frac{\chi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$ is only incurred in the production of new capital when using the final consumption good as the input, where $f(1) = f'(1) = 0$, $f''(1) > 0$ and $\chi > 0$. This specification of the adjustment cost implies that fewer units of new capital would be produced from one unit of investment whenever $\frac{i_t}{i_{t-1}}$ deviates from its steady state value 1 and the parameter χ reflects the magnitude of the cost.

Hence, the representative capital producer chooses the net investment level i_t to maximize the sum of the expected discounted future profits made from the sales revenue of new capital $q_t k_t$ net of the input cost $[q_t(1-\delta)k_{t-1} + i_t]$ and the investment adjustment cost $f\left(\frac{i_t}{i_{t-1}}\right) i_t$:

$$E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[q_t k_t - q_t(1-\delta)k_{t-1} - i_t - \frac{\chi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t \right] \tag{25}$$

where $\Lambda_{t,t+s} \equiv \beta^s \frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor, assuming households own the capital producers. Using $k_t = (1-\delta)k_{t-1} + i_t$, (25) can be simplified to:

$$E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[(q_t - 1)i_t - \frac{\chi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t \right] \tag{26}$$

[10] In a standard RBC model, the price of physical capital relative to consumption is 1. q_t is an important variable when introducing the agency problem in Section 4, where the cyclical variation in q_t can affect the entrepreneur's collateral value or net worth.

Taking the first order condition with respect to investment i_t gives the following expression for the real price of capital:

$$q_t = 1 + \frac{\chi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 + \chi\frac{i_t}{i_{t-1}}\left(\frac{i_t}{i_{t-1}} - 1\right) - \chi E_t\Lambda_{t,t+1}\left(\frac{i_{t+1}}{i_t}\right)^2\left(\frac{i_{t+1}}{i_t} - 1\right) \tag{27}$$

In the steady state, the real price of capital q is 1, since $i_{t+1} = i_t = i_{t-1}$. Any real profits Π_t^{CP} (which only arise outside the steady state) are rebated to the households, where $\Pi_t^{CP} = (q_t - 1)i_t - \frac{\chi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t$.

3.1.4 Retailers

Following Bernanke, Gertler and Gilchrist (1999), retailers are assumed to be monopolistically competitive. A continuum of mass 1 retailers, indexed by j, buy the wholesale good at a nominal price $p_{w,t}$ from entrepreneurs and use it as the only input to produce differentiated retail goods costlessly. Assume that one unit of the wholesale good can produce one unit of the differentiated product, so the marginal cost of production is the real price of the wholesale good $\frac{p_{w,t}}{p_t}$. Each retailer j produces a different variety $y_t(j)$ and charges a nominal price $p_t(j)$ for the differentiated product. The output of the final consumption good y_t is a CES composite of all the different varieties produced by the retailers, using the Dixit and Stiglitz (1977) framework:

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{28}$$

where $\epsilon > 1$ is the elasticity of intratemporal substitution between different varieties. Given the aggregate output index y_t, it can be calculated from the cost minimization problem of the buyers of the final consumption good that each retailer j faces a downward-sloping demand curve:

$$y_t(j) = \left[\frac{p_t(j)}{p_t}\right]^{-\epsilon} y_t \tag{29}$$

It can be shown that the aggregate consumption-based price index is:

$$p_t = \left[\int_0^1 p_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} \tag{30}$$

which is defined as the minimum expenditure to obtain one unit of consumption y_t in the cost minimization problem for the final output users.

Under monopolistic competition, retailers have price setting power, which is essential for introducing the nominal price rigidity à la Calvo (1983). With a nominal rigidity, monetary policy has real effects and the impact of a monetary policy shock can be analysed. Each retailer j sets its own price $p_t(j)$ taking the aggregate price p_t and the demand curve (29) as given. Under Calvo pricing, each retailer j is only allowed to change its price $p_t(j)$ in period t with probability $(1-\theta)$. The probability of price adjustment is independent of the time since the last adjustment, so in each period, a fraction $(1-\theta)$ of retailers reset their prices, whereas a fraction θ of retailers keep their prices fixed. Hence, $\theta \in (0,1)$ reflects the degree of price stickiness. Let $p_t^*(j)$ denote the optimal reset price in period t, then the corresponding demand facing retailer j who adjusted its price in period t, but cannot adjust its price in period $t+s$ is:

$$y_{t+s}^*(j) = \left[\frac{p_t^*(j)}{p_{t+s}}\right]^{-\epsilon} y_{t+s} \tag{31}$$

Retailer j chooses $p_t^*(j)$ to maximize the expected discounted value of real profits while its price is kept fixed at $p_t^*(j)$:

$$\sum_{s=0}^{\infty} \theta^s E_t \left\{ \Lambda_{t,t+s} \left[\frac{p_t^*(j)}{p_{t+s}} y_{t+s}^*(j) - \frac{1}{x_{t+s}} y_{t+s}^*(j) \right] \right\} \tag{32}$$

subject to the demand function (31). $\Lambda_{t,t+s} \equiv \beta^s \frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor, assuming households own the retailers. θ^s is the probability that $p_t^*(j)$ would remain fixed for s periods. $\frac{1}{x_{t+s}} = \frac{p_{w,t+s}}{p_{t+s}}$ is the price of the wholesale good in terms of the consumption units or the real marginal cost of production in period $t+s$. Taking the first order condition to solve for $p_t^*(j)$ gives the following optimal pricing equation:

$$p_t^*(j) = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{s=0}^{\infty} (\beta\theta)^s E_t u'(c_{t+s}) x_{t+s}^{-1} p_{t+s}^{\epsilon} y_{t+s}}{\sum_{s=0}^{\infty} (\beta\theta)^s E_t u'(c_{t+s}) p_{t+s}^{\epsilon-1} y_{t+s}} \tag{33}$$

The derivation is shown in Appendix A.1. In a symmetric equilibrium, all the retailers that adjust their prices in period t will set the same optimal price, such that $p_t^*(j) = p_t^*$. It is proved in Appendix A.2 that the aggregate price level evolves as follows:

$$p_t^{1-\epsilon} = \theta p_{t-1}^{1-\epsilon} + (1-\theta)(p_t^*)^{1-\epsilon} \tag{34}$$

which is independent of the heterogeneity of the retailers due to the convenience of the Calvo assumption. With randomly chosen price-adjusting retailers and the large number of retailers, there is no need to keep track of each retailer's price evolution.

Since there is a one-to-one conversion rate from the wholesale good to the differentiated retail good, in equilibrium the supply of wholesale good output $y_{w,t}$ is equal to the demand $y_t(j)$ over the entire unit interval of retailers j. Using retailer $j's$ individual demand function

(29), the wholesale good output can be expressed as:

$$y_{w,t} = \int_0^1 y_t(j)dj = y_t \int_0^1 \left[\frac{p_t(j)}{p_t}\right]^{-\epsilon} dj \tag{35}$$

As seen from the above equation, the final consumption good output y_t differs from the wholesale good output $y_{w,t}$ by a factor of the price dispersion $\int_0^1 \left[\frac{p_t(j)}{p_t}\right]^{-\epsilon} dj$. In a zero-inflation steady state, the price dispersion is one and the final output y_t would equal the wholesale good output $y_{w,t}$. Let $f_{3,t} \equiv \int_0^1 \left[\frac{p_t(j)}{p_t}\right]^{-\epsilon} dj$ denote the price dispersion, then the real profit Π_t^R made by the retailers is:

$$\Pi_t^R = \left(\frac{1}{f_{3,t}} - \frac{1}{x_t}\right) y_{w,t} = y_t - \frac{y_{w,t}}{x_t} \tag{36}$$

which will be rebated lump sum back to the households. The recursive formulation of the price dispersion used for numerical computation and the derivation for Π_t^R are shown in Appendix A.3.

3.1.5 Banking Sector

Assume there is a continuum of mass one banks, indexed by j, which are perfectly competitive with no price-setting power. Each bank j chooses the units of loans $b_t(j)$ and the units of deposits $d_t(j)$ to maximize the expected discounted value of real profits $\Pi_t^B(j)$, subject to the balance sheet identity (38) and the budget constraint in real terms (39). The gross nominal interest rate R_t is controlled by the central bank and is thus taken as given. Assume the banks are owned by the households, then $\Pi_t^B \equiv \sum_j \Pi_t^B(j)$ can be viewed as the dividends paid to the households. Each bank j maximizes the sum of expected discounted value of future dividends:

$$E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \Pi_t^B(j) \tag{37}$$

subject to the balance sheet identity:[11]

$$b_t(j) = d_t(j) \tag{38}$$

and the budget constraint, assuming costless financial intermediation:

$$\Pi_t^B(j) + b_{t-1}(j) + \frac{R_{t-1} d_{t-1}(j)}{\pi_t} = d_t(j) + \frac{R_{b,t-1} b_{t-1}(j)}{\pi_t} \tag{39}$$

[11]Following Andrés and Arce (2012), assume there is zero bank capital, so bank loans (assets) equal the deposits (liabilities).

It can be seen from (39) that in each period t, the total outflow of funds, consisting of the dividend payment to households $\Pi_t^B(j)$, loans granted to firms $b_t(j)$, and the gross real deposit interest payments to households $\frac{R_{t-1}d_{t-1}(j)}{\pi_t}$, would equal the total inflow of funds from the deposits saved by households $d_t(j)$ and the gross real loan interest payments received from firms $\frac{R_{b,t-1}b_{t-1}(j)}{\pi_t}$. Under perfect banking competition, the gross nominal loan rate $R_{b,t}$ is market-determined and equals R_t, which means the nominal loan interest margin $(R_{b,t} - R_t)$ is zero and each bank makes a zero profit in expectation. $\Pi_t^B(j)$ can be simplified by substituting the balance sheet identity (38) into the budget constraint (39):

$$\Pi_t^B(j) = \frac{1}{\pi_t}(R_{b,t-1} - R_{t-1})b_{t-1}(j) \tag{40}$$

Taking the first order condition of (37) with respect to $b_t(j)$ gives:

$$E_t\left[\Lambda_{t,t+1}\frac{1}{\pi_{t+1}}(R_{b,t} - R_t)\right] = 0 \tag{41}$$

Hence, under perfect banking competition, $R_{b,t} = R_t$ and the nominal loan margin is zero.

3.1.6 Central Bank

Suppose monetary policy is implemented by a Taylor rule with interest rate smoothing, which responds to both the level deviation of the gross inflation rate from the steady state inflation target π and the log deviation of output from its steady state y. The central bank controls the gross nominal interest rate on bank deposits or risk-free bonds, R_t, following the Taylor rule specification below:

$$R_t = (1-\rho_r)R + \rho_r R_{t-1} + (1-\rho_r)\left[\kappa_\pi(\pi_t - \pi) + \kappa_y ln\left(\frac{y_t}{y}\right)\right] + e_{r,t} \tag{42}$$

where variables without the time subscript represent steady state values and $e_{r,t}$ is a monetary policy shock which is a white noise process with zero mean and standard deviation σ_r. $\rho_r \in (0,1)$ is the interest rate smoothing parameter. κ_π and κ_y are non-negative weighting coefficients that reflect the central bank's relative preference for achieving the inflation target and minimizing output fluctuations. According to the Taylor principle, assume $\kappa_\pi > 1$ to ensure the nominal interest rate R_t is raised sufficiently in response to an increase in the gross inflation rate π_t so that the real interest rate rises. Due to interest rate smoothing, this policy rule implies a partial adjustment of R_t. As can be seen from (42), R_t is a convex combination of the lagged nominal interest rate R_{t-1} and the current target rate, which depends positively on the deviation of inflation from the steady state target and the log deviation of output from its steady state value. Let $R_{r,t}$ denote the gross real interest rate, then the relation between the nominal and real interest rates is given by the Fisher equation:

$$R_{r,t} = E_t \frac{R_t}{\pi_{t+1}} \tag{43}$$

3.2 Imperfect Banking Competition

Imperfect banking competition is analysed by replacing the perfectly competitive banking sector by a Cournot banking sector. The model set-up is unchanged apart from the banking sector which is now imperfectly competitive. This implies that banks' quantity-setting decisions can affect the market loan rate in a Cournot equilibrium. Only the differences from Section 3.1 are discussed here. Assume now there are N banks in the economy, indexed by j, which operate under Cournot competition. Each bank j sets the quantity of loans $b_t(j)$ to maximize the sum of the present discounted value of future dividends:

$$E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \Pi_t^B(j) \tag{44}$$

where

$$\Pi_t^B(j) = \frac{1}{\pi_t} \left[R_{b,t-1} \left(b_{t-1}(j) + \sum_{m \neq j} b_{t-1}(m) \right) - R_{t-1} \right] b_{t-1}(j) \tag{45}$$

taking the quantities of loans chosen by the other banks $m \neq j$ as given. The real profit $\Pi_t^B(j)$ is positive and will be rebated back to the households. A key difference from Section 3.1.5 is that $R_{b,t}$ now represents the inverse loan demand function, which is a decreasing function of b_t. This is crucial for introducing imperfect banking competition. $R_{b,t}$ as a function of $b_t(j)$ means that each bank j has some control over the equilibrium gross loan interest rate by altering its own quantity of loans given the other banks' loan quantities and this is taken into consideration by the bank j under Cournot competition when choosing $b_t(j)$. Solving the profit maximization problem with respect to $b_t(j)$ gives the following first order condition:

$$E_t \left\{ \Lambda_{t,t+1} \frac{1}{\pi_{t+1}} \left[\frac{\partial R_{b,t}}{\partial b_t(j)} b_t(j) + R_{b,t} - R_t \right] \right\} = 0 \tag{46}$$

In a Cournot equilibrium, total optimal loan quantity is $b_t = b_t(j) + \sum_{m \neq j} b_t(m)$ and each bank produces a share of the total quantity. Assuming banks are identical, then $b_t(j) = \frac{b_t}{N}$ in equilibrium. Since $\frac{\partial R_{b,t}}{\partial b_t(j)} = \frac{\partial R_{b,t}}{\partial b_t} \frac{\partial b_t}{\partial b_t(j)} = \frac{\partial R_{b,t}}{\partial b_t}$, in Cournot equilibrium, the first order condition (46) can be rewritten as:

$$E_t\left\{\Lambda_{t,t+1}\frac{1}{\pi_{t+1}}\left[\frac{\partial R_{b,t}}{\partial b_t}\frac{b_t}{N}+R_{b,t}-R_t\right]\right\}=0 \tag{47}$$

where the market loan demand b_t and the partial derivative $\frac{\partial R_{b,t}}{\partial b_t}$ can be calculated explicitly from the representative entrepreneur's problem in Section 3.1.2.[12] The market loan demand is given by the binding collateral constraint:

$$b_t=m_{h,t}E_t\left[\frac{q_{h,t+1}h_t^E\pi_{t+1}}{R_{b,t}}\right]+m_{k,t}E_t\left[\frac{q_{t+1}k_t(1-\delta)\pi_{t+1}}{R_{b,t}}\right] \tag{48}$$

As can be seen from (48), $R_{b,t}$ has a direct negative effect on market loan demand b_t since an increase in $R_{b,t}$ can reduce the entrepreneur's borrowing capacity. Besides, $R_{b,t}$ also has an indirect effect on b_t by influencing the entrepreneur's demand for housing and physical capital, which can be seen from the first order conditions for housing (19) and physical capital (18). Hence, when bank j chooses $b_t(j)$ that could affect the equilibrium gross loan rate $R_{b,t}$ under Cournot competition, it would need to consider how the entrepreneurs would respond by changing their demand for physical capital $\frac{\partial k_t}{\partial R_{b,t}}$ and for housing $\frac{\partial h_t^E}{\partial R_{b,t}}$. The real price of housing $q_{h,t+1}$ and the real price of physical capital q_{t+1} in period $t+1$ are determined in equilibrium, so they are independent of $R_{b,t}$. Taking the total derivative of b_t with respect to $R_{b,t}$ gives:

$$\begin{aligned}\frac{\partial b_t}{\partial R_{b,t}}=&-R_{b,t}^{-2}\left\{m_{h,t}E_t\left[q_{h,t+1}h_t^E\pi_{t+1}\right]+m_{k,t}E_t\left[q_{t+1}k_t(1-\delta)\pi_{t+1}\right]\right\}\\&+m_{h,t}E_t\left[\frac{q_{h,t+1}\pi_{t+1}}{R_{b,t}}\right]\frac{\partial h_t^E}{\partial R_{b,t}}+m_{k,t}E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}}\right]\frac{\partial k_t}{\partial R_{b,t}}\end{aligned} \tag{49}$$

which can be simplified using the binding borrowing constraint (48):

$$\frac{\partial b_t}{\partial R_{b,t}}=-\frac{b_t}{R_{b,t}}+m_{h,t}E_t\left[\frac{q_{h,t+1}\pi_{t+1}}{R_{b,t}}\right]\frac{\partial h_t^E}{\partial R_{b,t}}+m_{k,t}E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}}\right]\frac{\partial k_t}{\partial R_{b,t}} \tag{50}$$

The partial derivatives $\frac{\partial h_t^E}{\partial R_{b,t}}$ and $\frac{\partial k_t}{\partial R_{b,t}}$ can be calculated from the first order conditions (19) and (18), which is shown in Appendix C. $\frac{\partial R_{b,t}}{\partial b_t}$ can then be calculated as $\left(\frac{\partial b_t}{\partial R_{b,t}}\right)^{-1}$.

Under perfect banking competition, each bank faces a perfectly elastic loan demand,

[12] In equilibrium, the total supply of loans from the Cournot banking sector equals the total market loan demand derived from the entrepreneur's problem.

although the market loan demand is downward-sloping and is the same as (48). In the Cournot case, each bank faces the loan demand $\frac{b_t}{N}$ in equilibrium. Define the price elasticity of the loan demand PED_t facing bank j as:

$$PED_t \equiv -\frac{\partial b_t(j)}{\partial R_{b,t}}\frac{R_{b,t}}{b_t(j)} = -\frac{\partial b_t}{\partial R_{b,t}}\frac{NR_{b,t}}{b_t} \tag{51}$$

where the second equality holds in a Cournot equilibrium. An increase in PED_t indicates that bank j faces a more elastic loan demand. It is shown in (52) that PED_t increases in the number of banks N, the amount of borrowing secured by housing collateral relative to the total borrowing $\frac{b_{h,t}}{b_t}$, the proportion of borrowing secured by physical collateral $\frac{b_{k,t}}{b_t}$, and decreases in the value of housing relative to the maximum amount of borrowing against the housing collateral $\frac{q_{h,t}h_t^E}{b_{h,t}}$, the value of physical capital relative to the maximum amount of borrowing against the capital collateral $\frac{q_t k_t}{b_{k,t}}$, the entrepreneur's consumption c_t^E and the Lagrange multiplier associated with the collateral constraint $\lambda_{2,t}^E$.

$$PED_t = N\left(1 + \frac{\frac{b_{h,t}}{b_t} + \frac{v}{1-v-\alpha}}{\frac{q_{h,t}h_t^E}{b_{h,t}} + (\frac{1}{m_{h,t}} - 1)\lambda_{2,t}^E c_t^E - \frac{1}{m_{h,t}}} + \frac{\frac{\alpha}{1-v-\alpha} + \frac{b_{k,t}}{b_t}}{\frac{q_t k_t}{b_{k,t}} + (\frac{1}{m_{k,t}} - 1)\lambda_{2,t}^E c_t^E - \frac{1}{m_{k,t}}}\right) \tag{52}$$

where $b_{h,t} = m_{h,t}E_t\left[\frac{q_{h,t+1}h_t^E\pi_{t+1}}{R_{b,t}}\right]$ and $b_{k,t} = m_{k,t}E_t\left[\frac{q_{t+1}k_t(1-\delta)\pi_{t+1}}{R_{b,t}}\right]$. The derivation for (52) is shown in Appendix D.

As can be seen from (52), an increase in N makes the loan demand facing each bank more elastic because the total market loan demand is shared across more banks, reflecting less market power possessed by each bank. After the negative shocks, lower $\frac{b_{h,t}}{b_t}$ and $\frac{b_{k,t}}{b_t}$ indicate that the percentage fall in $b_{h,t}$ or $b_{k,t}$ from its steady state is larger than the percentage fall in b_t from its steady state, implying that the entrepreneur is more constrained due to a reduction in the borrowing capacity and thus resulting in a more inelastic loan demand. A rise in $\frac{q_{h,t}h_t^E}{b_{h,t}}$ and $\frac{q_t k_t}{b_{k,t}}$ also makes the loan demand more inelastic. This is because a larger fall in $b_{h,t}$ relative to $q_{h,t}h_t^E$ reflects a fall in the leverage ratio associated with the housing only $\frac{b_{h,t}}{q_{h,t}h_t^E}$. Deleveraging indicates that the entrepreneur is less sensitive to changes in the collateral constraints and hence the loan demand is more inelastic. A higher $\lambda_{2,t}^E$ means that the borrowing constraint binds more tightly and entrepreneurs are more financially constrained, resulting in a more inelastic loan demand.

Substituting the definition for PED_t (52) into banks' first order condition (47), it can be shown that a higher price elasticity of loan demand directly reduces $R_{b,t}$ since

$$R_{b,t} = \frac{E_t \Lambda_{t,t+1} \frac{1}{\pi_{t+1}} R_t}{E_t \Lambda_{t,t+1} \frac{1}{\pi_{t+1}} (1 - PED_t^{-1})} \tag{53}$$

Under Cournot competition, banks with market power can affect the equilibrium loan rate by taking advantage of the endogenously changing loan demand elasticity.

3.3 Equilibrium Conditions

In equilibrium, the aggregate resource constraint is:

$$c_t + c_t^E + i_t + \frac{\chi}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 i_t = y_t \tag{54}$$

which is also the goods market clearing condition. Let b_t^B and d_t^B denote the total units of loans given out and deposits taken in by the banking sector respectively. Under perfect banking competition with a continuum of unit mass banks, $b_t^B = \int_0^1 b_t(j)dj$ and $d_t^B = \int_0^1 d_t(j)dj$, while under a Cournot banking sector, $b_t^B = \sum_{j=1}^N b_t(j)$ and $d_t^B = \sum_{j=1}^N d_t(j)$ in equilibrium. The other market clearing conditions are: (labor) $l_t = l_t^E$, (capital) $b_t = q_t k_t + q_{h,t} h_t^E - \beta^E n w_t$, (housing) $h_t + h_t^E = 1$, where the total fixed supply of housing is normalised to 1, (deposits) $d_t = d_t^B$, (loans) $b_t = b_t^B$, and (financial intermediation) $b_t^B = d_t^B$.

4 Calibration

The model with two types of banking competition, are solved numerically using Dynare after calibrating the parameters to a quarterly frequency. The household subjective discount factor β, is set at 0.995, giving an annualised net real deposit rate of $\left(\frac{1}{0.995} - 1\right) * 4 \approx 2\%$. The subjective discount factor for the entrepreneur $\beta^E = 0.97$ is taken from Andrés and Arce (2012). As shown in Section 3.1.2, the subjective discount factor for entrepreneurs β^E needs to be smaller than β, to ensure a binding collateral constraint in the steady state. When imperfect banking competition is introduced into the framework of collateral constraints, the restriction to ensure a binding borrowing constraint is no longer $\beta^E < \beta$, because the loan interest margin is greater than zero. Since $\lambda_2^E = \frac{1}{c^E}\left(1 - \beta^E \frac{R_b}{\pi}\right)$ and $\frac{R}{\pi} = \frac{1}{\beta}$, as shown by equation (20) and (7), $\lambda_2^E = \frac{1}{c^E}\left(1 - \frac{\beta^E}{\beta}\frac{R_b}{R}\right)$. Hence, as long as $\frac{R_b}{R} < \frac{\beta}{\beta^E}$ under imperfect banking competition, λ_2^E will be positive, which means the borrowing constraint will bind in the steady state (Andrés and Arce, 2012). As a result, when replacing the perfectly competitive banking sector with the Cournot banking sector, the ratio of gross nominal loan rate to gross nominal deposit rate in the steady state $\frac{R_b}{R}$ must satisfy the following condition:

$$1 \leqslant \frac{R_b}{R} < \frac{\beta}{\beta^E} \tag{55}$$

where the first inequality comes from the nonnegativity of bank profits. The upper bound $\frac{\beta}{\beta^E}$ imposes a limit on the size of the loan margin.

Standard parameters such as the physical capital share α, the depreciation rate δ, and the elasticity of substitution among differentiated retail goods ϵ are chosen to be 0.33, 0.025, and 6 respectively. The steady state gross inflation target π is set at 1. In this zero-inflation steady state, $\epsilon = 6$ implies a final good price markup over the wholesale good of 20% since $x = \frac{\epsilon}{\epsilon - 1}$. Given the above parameters, the relative utility weight on leisure time ϕ is set at 1.45 to achieve steady state labor hours l of around 0.33. The relative utility weight on the holdings of the real estate for the households $\phi_h = 0.1$ and the real estate share of the wholesale good output $v = 0.05$ are taken from Andrés and Arce (2012) to achieve $h^E = 0.22$ under perfect banking competition in the steady state. The investment adjustment cost parameter χ would not affect the steady state and is set at 1.7, following Gertler and Karadi (2011), since the set-up for the capital producers is similar to their paper. The probability θ of retailers keeping prices fixed in each period is set at 0.75, resulting in a price rigidity of $\frac{1}{1-0.75} = 4$ quarters on average. The parameters in the Taylor rule and the shock-related parameters are within the range considered in the literature (Bonciani and Van Roye, 2015; Gertler and Karadi, 2011; Iacoviello, 2005). Specifically, the interest rate smoothing parameter ρ_r, the feedback coefficients on inflation κ_π and output κ_y, are specified to be 0.8, 1.5 and 0.03 respectively. The persistence parameters of the productivity shock ψ, the loan to value ratio shocks ψ_{m_h} and ψ_{m_k} are 0.97, 0.8 and 0.8 respectively. Besides, the standard deviation of the monetary policy shock σ_r is 0.001 and the standard deviations of the productivity shock σ_z, the loan-to-value ratio shocks σ_{m_h} and σ_{m_k} are all set to be 0.01.

The loan-to-value ratios for housing and physical capital are chosen to be $m_h = 0.8$ and $m_k = 0.5$ respectively. This is because real estate tends to be a better collateral than physical capital because it has a higher resale value. The results are robust to different values of m_h and m_k. Given $m_h = 0.8$, $m_k = 0.5$, $\beta = 0.995$, $\beta^E = 0.97$, $\alpha = 0.33$, $v = 0.05$ and $\delta = 0.025$, the number of banks N is set at 2 to achieve $R_b = 1.02651$ and a steady state real loan margin of 215 basis points. It can be proved that the condition $\frac{R_b}{R} \in [1, \frac{\beta}{\beta^E})$ is satisfied and the borrowing constraint is binding in the steady state with a Cournot banking sector. A summary of the calibrated parameters is shown in Table 1 in Appendix G. Given the calibration in this section, the steady state values of some key variables are summarised in Table 2 in Appendix G.

5 Dynamic Analysis

The impulse responses of some key variables under the two types of banking competition are compared after a contractionary monetary policy shock, a negative productivity shock and negative shocks to the loan-to-value ratios $m_{k,t}$ and $m_{h,t}$. The model is solved numerically using Dynare,[13] in which the impulse response functions (IRFs) are computed as the deviation in the trajectory of a variable from its steady state value following a shock at the beginning of period 1. The nonlinear model is solved by log-linearization around the steady state and the responses of variables are expressed as percentage deviations from the steady state in order to compare impulse responses from different models with different steady states.[14] All the equations used to compute the model are shown in Appendix E.

5.1 Monetary Policy Shock

An unexpected one-time monetary policy shock is implemented by a 1 standard deviation ($\sigma_r = 0.001$) or 10 basis points increase in the white noise term $e_{r,t}$ in the Taylor rule at the beginning of period 1 ($\sigma_{r,t} = 0.001$ in $t = 1$), so that R_t is raised. As can be seen from Figure 1, after a contractionary monetary policy shock, the responses of output, consumption, investment and physical capital tend to be amplified under imperfect banking competition. Following Andrés and Arce (2012), there are three relevant channels through which a monetary shock can affect this model economy: the traditional interest rate channel, the endogenous loan interest margin, and the net worth effect.

The interest rate channel works in a standard way via sticky prices. When a contractionary monetary shock raises the gross nominal interest rate R_t, the gross real interest rate will also rise due to price rigidity. The increase in the real deposit rate reduces consumption via household intertemporal substitution. However, there is little difference in the response of the real deposit rate for the two types of banking competition apart from the impact change, indicating that this channel is not very important in explaining the amplified responses of output and consumption under imperfect banking competition.

Under perfect banking competition, the loan interest margin is zero, hence households and entrepreneurs face the same real interest rate. A rise in real interest rate would reduce the investment in physical capital. It can be seen from Figure 1 that the maximum decrease in investment reaches around 1.5% under perfect banking competition, whereas the maximum

[13]The command 'stoch_simul(order=1)' is used, which solves the stochastic model using a first-order Taylor approximation of the decisions rules.

[14]Since Dynare only performs linearization, variables are log-transformed and then a linear Taylor approximation in logs is implemented. Let $\widehat{var}_t$ denote the log deviation of a variable var_t from its steady state value var, such that $\widehat{var}_t = ln(var_t) - ln(var)$.

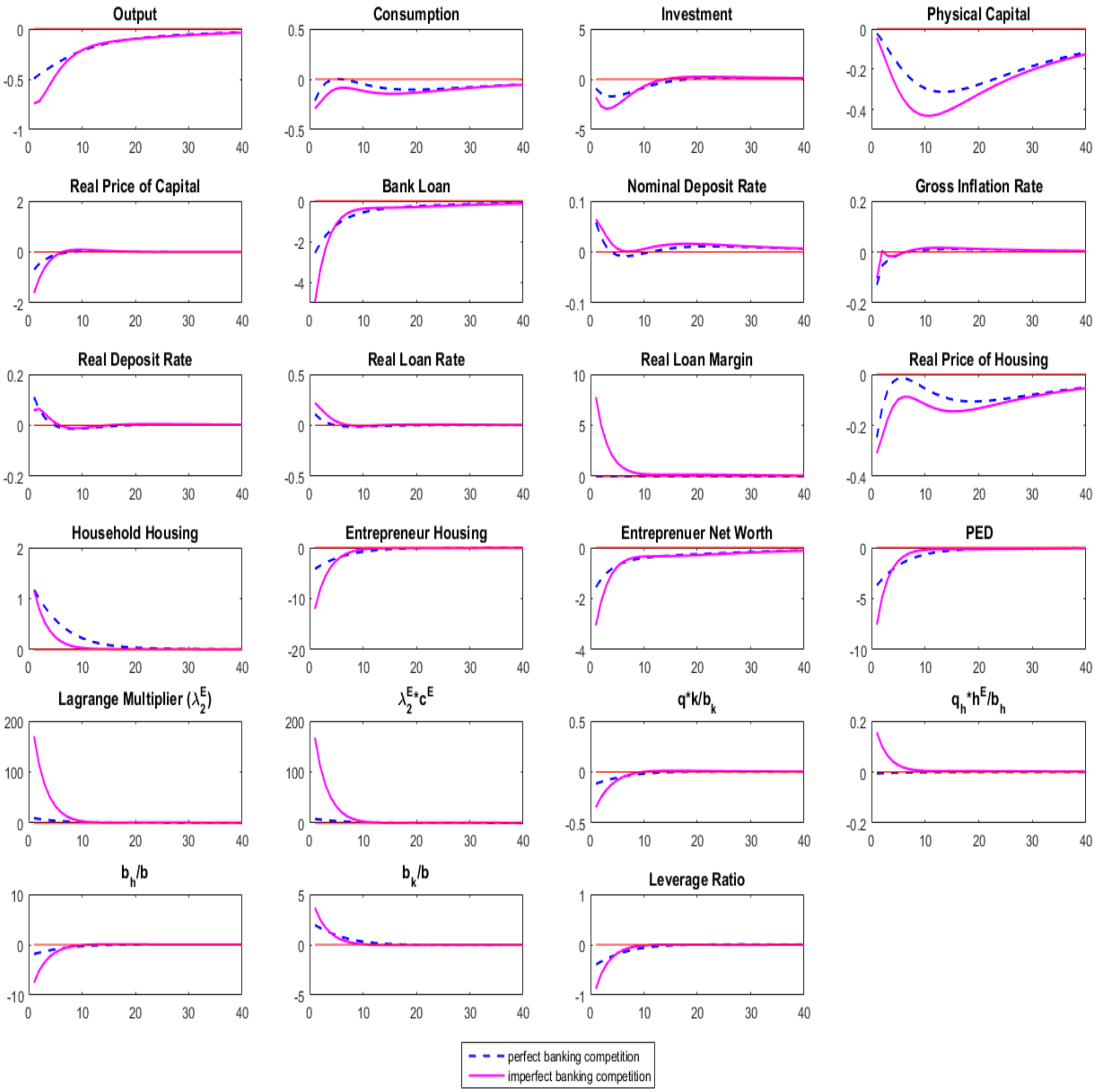

Figure 1: Impulse responses to a contractionary monetary shock

Note: Horizontal axis shows quarters after the shock that occurs at the beginning of period 1. Vertical axis shows the percentage deviation from the steady state. The blue dashed line corresponds to perfect banking competition, and the pink solid line corresponds to imperfect banking competition.

decline is 2.5% relative to the steady state under imperfect banking competition. This amplified response may be explained by the increase in the real loan margin. Although the real deposit rate rises by around 0.1% immediately after the shock due to sticky prices, with a Cournot banking sector, the impact increase of the real loan rate is around 0.23%. This significantly larger rise in the real loan rate compared to the rise in the real deposit rate under imperfect banking competition, as reflected by a large positive response of the real loan margin, could explain the amplified responses of investment and physical capital. Given the calibration in Section 4, the steady state loan margin is 215 basis points. Hence, the rise in the real loan margin by 7.5% implies that the real loan margin increases to $215 * (1 + 0.075) \approx 231$ basis points immediately after the shock.

The real loan margin rises on impact because of the joint effect between the time-varying loan demand elasticity PED_t and banks' market power. As can be seen from Figure 1, the elasticity of loan demand facing each bank falls on impact under Cournot banking competition, indicating the loan demand becomes more inelastic. Hence, each bank with a certain degree of market power can take advantage of this reduction in elasticity by reducing the quantity of loans to achieve a higher equilibrium loan rate. The response of PED_t under perfect competition refers to the percentage change in market loan demand elasticity relative to its steady state. From the expression for PED_t (52), the fall in loan demand elasticity on impact can be explained by an increase in the ratio of entrepreneurial housing value to the amount of borrowing secured by the housing collateral $\frac{q_{h,t}h_t^E}{b_{h,t}}$, an increase in the Lagrange multiplier $\lambda_{2,t}^E$ associated with the borrowing constraint, and a reduction in the ratio of borrowing secured by housing collateral to total borrowing $\frac{b_{h,t}}{b_t}$.

Intuitively, an increase in $\frac{q_{h,t}h_t^E}{b_{h,t}}$ makes the loan demand more inelastic because it implies that the leverage over the housing asset $\frac{b_{h,t}}{q_{h,t}h_t^E}$ decreases. A reduction in leverage means entrepreneurs are less sensitive to changes in collateral constraint and hence the loan demand is more inelastic. As shown in Figure 1, the Lagrange multiplier $\lambda_{2,t}^E$ associated with the borrowing constraint rises by around 175%, and this is because the real loan rate increases and asset prices fall after a contractionary monetary shock.[15] From the binding borrowing constraint (48), these changes would lead to a more tightly binding borrowing constraint, indicating a reduction in the entrepreneur's borrowing capacity, and hence a more inelastic loan demand. A reduction in the ratio of borrowing secured by housing collateral to total borrowing $\frac{b_{h,t}}{b_t}$ can also indicate a more inelastic loan demand because when the fall in $b_{h,t}$ is larger than the fall in b_t, it implies that there is a reduction in borrowing capacity and the entrepreneurs are more constrained, assuming $\frac{b_{k,t}}{b_t}$ is unchanged, and hence the loan demand

[15]Under imperfect banking competition, $\lambda_2^E = 0.071994$, and under perfect banking competition, $\lambda_2^E = 0.247924$. Only negative shocks are examined in this paper to ensure that $\lambda_{2,t}^E$ is always positive.

is more inelastic. Although the ratio of borrowing secured by physical capital collateral to total borrowing $\frac{b_{k,t}}{b_t}$ rises on impact and the ratio of physical capital value to the amount of borrowing secured by the physical capital $\frac{q_t k_t}{b_{k,t}}$ falls on impact, which tend to raise PED_t, these two effects are dominated by the downward pressure on PED_t. As can be seen from Figure 1, the impact rise in $\frac{b_{k,t}}{b_t}$ is around 4% and the impact fall in $\frac{q_t k_t}{b_{k,t}}$ is less than 0.35%, whereas the impact rise in $\lambda^E_{2,t}$ is around 175%, which appears to be the main driving force behind the impact fall in loan demand elasticity.

The net worth effect can be shown by loglinearizing the expression for nw_t (21). It is shown in Appendix F that

$$\begin{aligned}\widehat{nw_t} =& \frac{\beta^E}{\Omega^{-1}-1}\Bigg\{\frac{(\alpha+v)y_w}{xb}(\widehat{y_{w,t}}-\widehat{x}_t)+\frac{qk}{b}(1-\delta)[(1-m_k)(\widehat{q}_t+\widehat{k_{t-1}})-m_k\widehat{m_{k,t}}] \\ &+\frac{q_h h^E}{b}[(1-m_h)(\widehat{q_{h,t}}+\widehat{h^E_{t-1}})-m_h\widehat{m_{h,t}}]\Bigg\}\end{aligned} \tag{56}$$

where $\Omega \equiv \frac{b}{q_h h^E + qk}$ is the steady state leverage ratio, which is defined as the amount of borrowing against the total value of the collateral assets. Use the binding borrowing constraint (48) to rewrite Ω as:

$$\Omega = \frac{m_h \pi q_h h^E + m_k \pi q k(1-\delta)}{R_b(q_h h^E + qk)} \tag{57}$$

Since R_b tends to be lower and thus $\frac{m_h \pi q_h h^E + m_k \pi q k(1-\delta)}{(q_h h^E + qk)}$ tends to be higher under perfect banking competition than under imperfect banking competition, Ω is likely to be higher under perfect banking competition.[16] As shown in (56), a lower steady state leverage ratio under imperfect banking competition tends to attenuate the initial negative effect on the entrepreneur's net worth due to a contractionary monetary shock, taking everything else as given. However, to determine if the net worth is attenuated under imperfect banking competition relative to the perfect banking competition, it is also necessary to look at the values of $\frac{y_w}{b}$, $\frac{qk}{b}$ and $\frac{q_h h^E}{b}$ and the responses of variables $(\widehat{y_{w,t}}-\widehat{x}_t)$, $(\widehat{q}_t+\widehat{k_{t-1}})$ and $(\widehat{q_{h,t}}+\widehat{h^E_{t-1}})$ under the two types of banking competition. Given the calibration in this paper, $\frac{y_w}{b}$, $\frac{qk}{b}$ and $\frac{q_h h^E}{b}$ are larger under imperfect banking competition. However, the responses of $(\widehat{y_{w,t}}-\widehat{x}_t)$, $(\widehat{q}_t+\widehat{k_{t-1}})$ and $(\widehat{q_{h,t}}+\widehat{h^E_{t-1}})$ under the two types of banking competition can also differ. Hence, the effect of imperfect banking competition via the net worth channel is ambiguous.

After the contractionary monetary shock, it is shown in Figure 1 that imperfect banking competition amplifies $\widehat{nw_t}$ and this is likely because of the much larger $(\widehat{q}_t+\widehat{k_{t-1}})$ and $(\widehat{q_{h,t}}+\widehat{h^E_{t-1}})$ under imperfect banking competition. Given the multipliers $\frac{qk}{b}$ and $\frac{q_h h^E}{b}$ are also

[16] This can be verified using the steady state values shown in Table 2 in Appendix G. Under perfect banking competition $\Omega = 0.606093$, whereas it is 0.544574 under imperfect banking competition.

larger under imperfect banking competition, in this case, the response of nw_t is amplified under imperfect banking competition, as can be seen in Figure 1. When the contractionary monetary shock adversely affects the entrepreneur's net worth, limiting their access to external financing, their demand for housing and physical capital would fall. This fall in demand would depress the real prices of housing and physical capital, further reducing their net worth and hence their access to external financing, which leads to a further fall in demand. After a contractionary monetary shock, this net worth effect tends to amplify the response of output under imperfect banking competition and reinforce the amplification effect arising from the countercyclical real loan margin.

5.2 Productivity Shock

After a one standard deviation ($\sigma_z = 0.01$) or 1% negative productivity shock at the beginning of period 1 ($\sigma_{z,t} = 0.01$ in $t = 1$), the responses of output, investment and physical capital are slightly attenuated under imperfect banking competition, in spite of the countercyclical real loan margin. There are two possible reasons to explain the absence of the amplification effect in this case. First, the positive response of the real loan margin is much smaller compared to that of the contractionary monetary shock.[17] Second, the net worth effect tends to attenuate the response of output in this case as the entrepreneur's net worth is attenuated after the negative productivity shock.

As discussed in Section 5.1, the impact rise in the real loan margin is due to the joint effect of the decreasing loan demand elasticity and bank's market power. The fall in loan demand elasticity is mainly driven by a more tightly binding borrowing constraint after a negative productivity shock, which can be seen from a positive response of $\lambda_{2,t}^E$ on impact in Figure 2. On the one hand, falling real asset prices increase the tightness of the binding borrowing constraint. On the other hand, after a negative productivity shock, the real loan rate falls due to higher inflationary pressure, as can be seen from Figure 2, which then improves the entrepreneur's borrowing capacity. By contrast, after a contractionary monetary shock, changes in the real loan rate and asset prices all work in the same direction in reducing the entrepreneur's borrowing capacity. As a result, after the negative productivity shock, the tightness of the binding borrowing constraint increases to a lesser extent, which can be seen by comparing the magnitude of the response of $\lambda_{2,t}^E$ in Figure 1 and 2. Besides, the impact reduction in $\frac{b_{h,t}}{b_t}$ and the impact rise in $\frac{q_{h,t}h_t^E}{b_{h,t}}$ are much smaller compared to their counterparts after the contractionary monetary shock. These all contribute to a smaller

[17] Although the size of the monetary shock differs from that of the productivity shock, the reasoning is unaffected because the size of the productivity shock is larger. For a 0.1% negative productivity shock, the response of the real loan margin is even smaller.

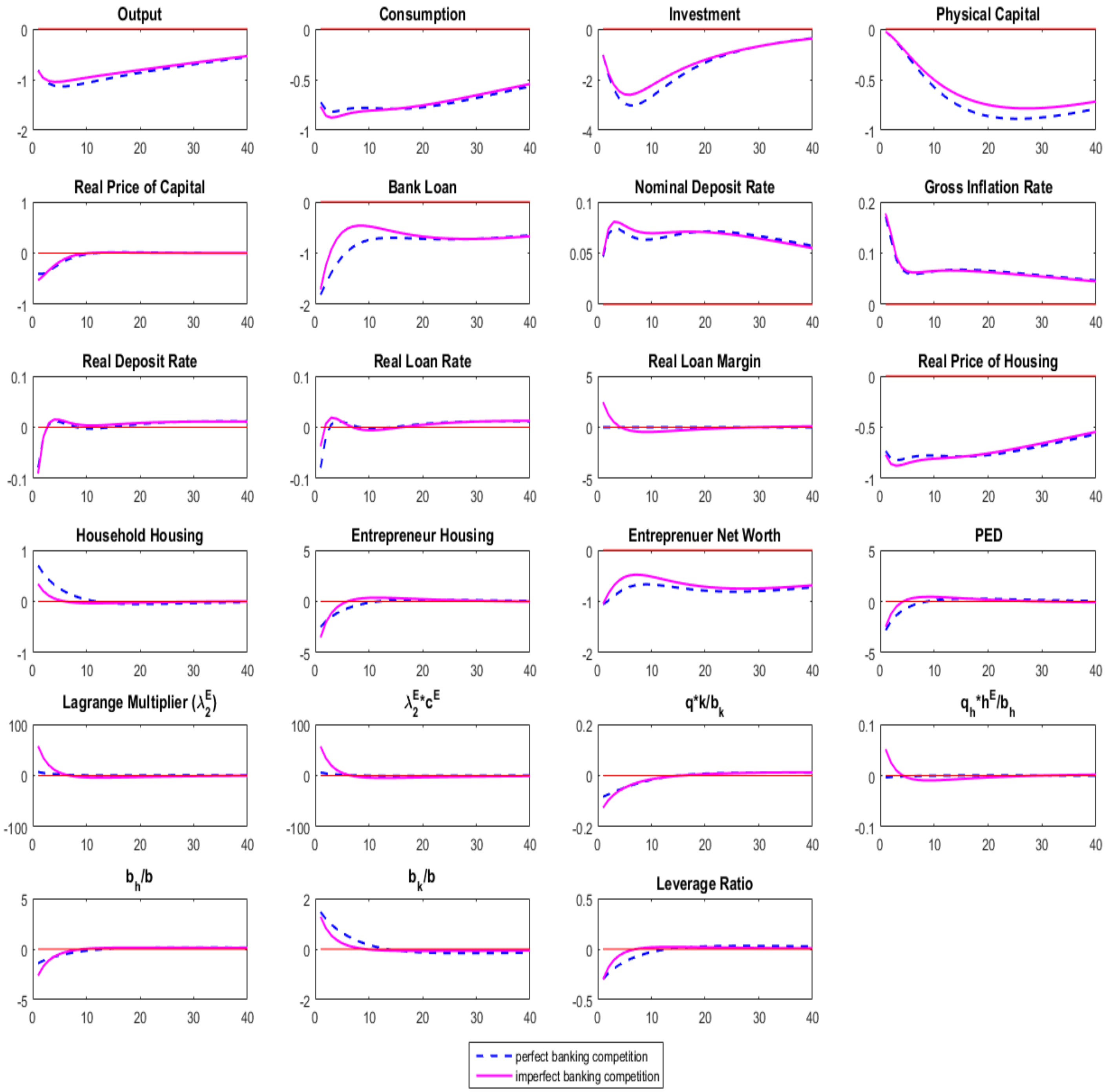

Figure 2: Impulse responses to a negative productivity shock

Note: Horizontal axis shows quarters after the shock that occurs at the beginning of period 1. Vertical axis shows the percentage deviation from the steady state. The blue dashed line corresponds to perfect banking competition, and the pink solid line corresponds to imperfect banking competition.

decrease in PED_t and hence a smaller rise in the real loan margin. As shown in Figure 2, the magnitude of the countercyclical real loan margin is much smaller compared to the contractionary monetary shock.

Figure 2 shows that the response of entrepreneur's net worth is attenuated under imperfect banking competition. According to the expression for the percentage deviation of nw_t from its steady state $\widehat{nw_t}$ (56), given that $\widehat{q_t}$ and $(\widehat{q_{h,t}} + \widehat{h^E_{t-1}})$ are very similar under the two types of banking competition, the attenuation effect of imperfect banking competition on the response of net worth and hence output is likely to be driven by the smaller response of k_t under imperfect banking competition. Hence, in this case, the net worth effect tends to attenuate the response of output and works in the opposite direction to the amplification effect from the countercyclical real loan margin.

5.3 Collateral Shocks

5.3.1 Negative Shock to $m_{h,t}$

After a one standard deviation ($\sigma_{m_h} = 0.01$) or 1% negative $m_{h,t}$ shock at the beginning of period 1 ($\sigma_{m_{h,t}} = 0.01$ in $t = 1$), the responses of output, investment and physical capital are all amplified under imperfect banking competition. Similar to the case of the contractionary monetary shock, the amplification effect can be explained by the countercyclical real loan margin and the net worth effect.

An exogenous shock that lowers the fraction $m_{h,t}$ (of the housing collateral that can be recouped by banks when the entrepreneurs fail to repay the debt) tends to reduce PED_t, meaning that the loan demand is more inelastic. Intuitively, a decrease in $m_{h,t}$ reduces the entrepreneur's borrowing capacity and makes the borrowing constraint bind more tightly, as reflected by an increase in $\lambda^E_{2,t}$ by around 90%. The positive response of $\lambda^E_{2,t}$ directly contributes to a fall in PED_t by around 5% on impact, raising the real loan rate and leading to an increase in the real loan margin for a given real deposit rate.

The net worth effect reinforces the amplification effect from the increase in real loan margin, because the fall in entrepreneur's net worth is larger under imperfect banking competition, as can be seen from Figure 3. The lower net worth constrains the entrepreneur's ability to borrow, reducing the demand for physical capital and housing and thus further reducing the asset prices. Hence, imperfect banking competition working through this net worth channel, by amplifying the entrepreneur's net worth, also tends to amplify the response of output.

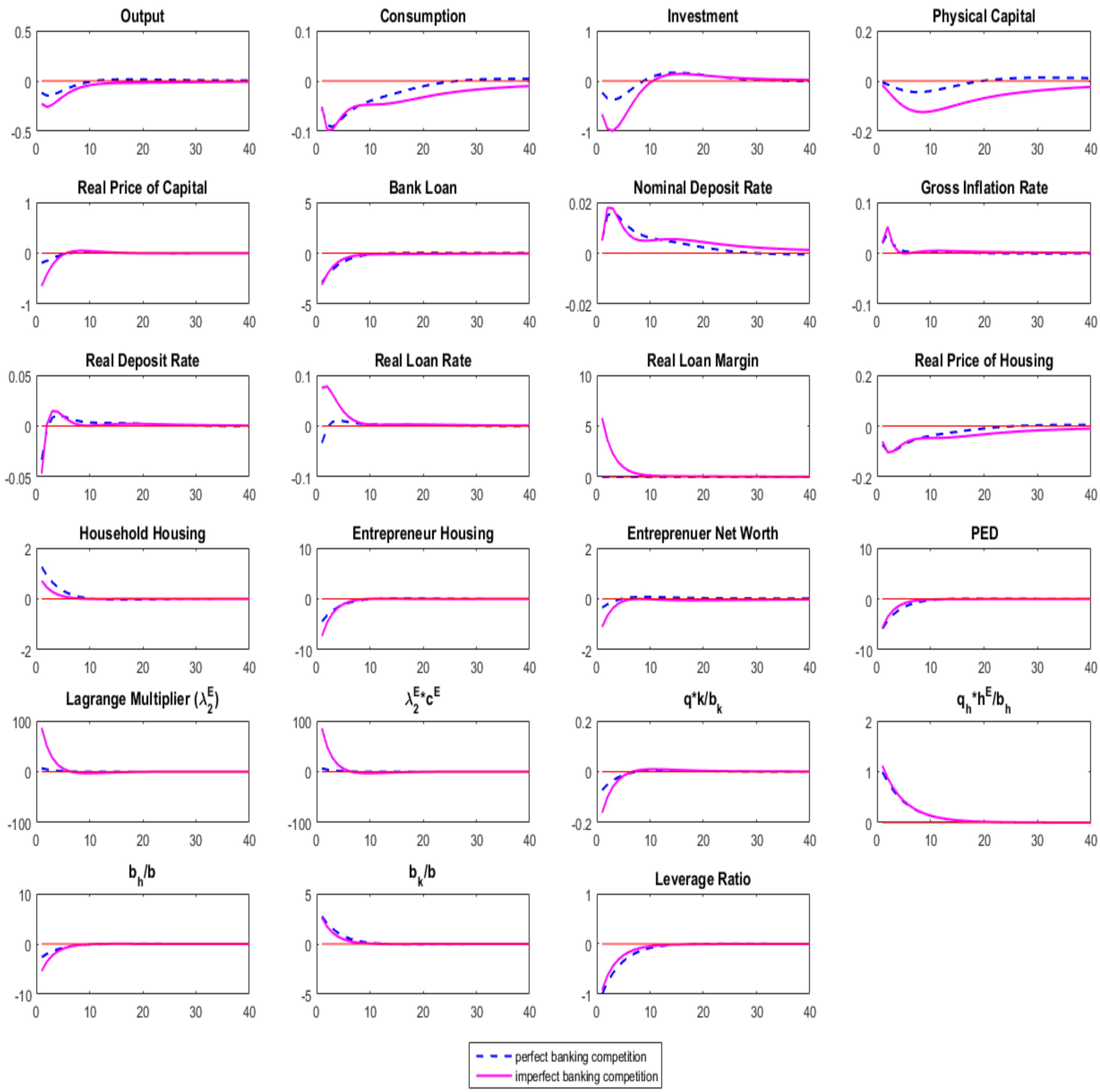

Figure 3: Impulse responses to a negative m_h shock

Note: Horizontal axis shows quarters after the shock that occurs at the beginning of period 1. Vertical axis shows the percentage deviation from the steady state. The blue dashed line corresponds to perfect banking competition, and the pink solid line corresponds to imperfect banking competition.

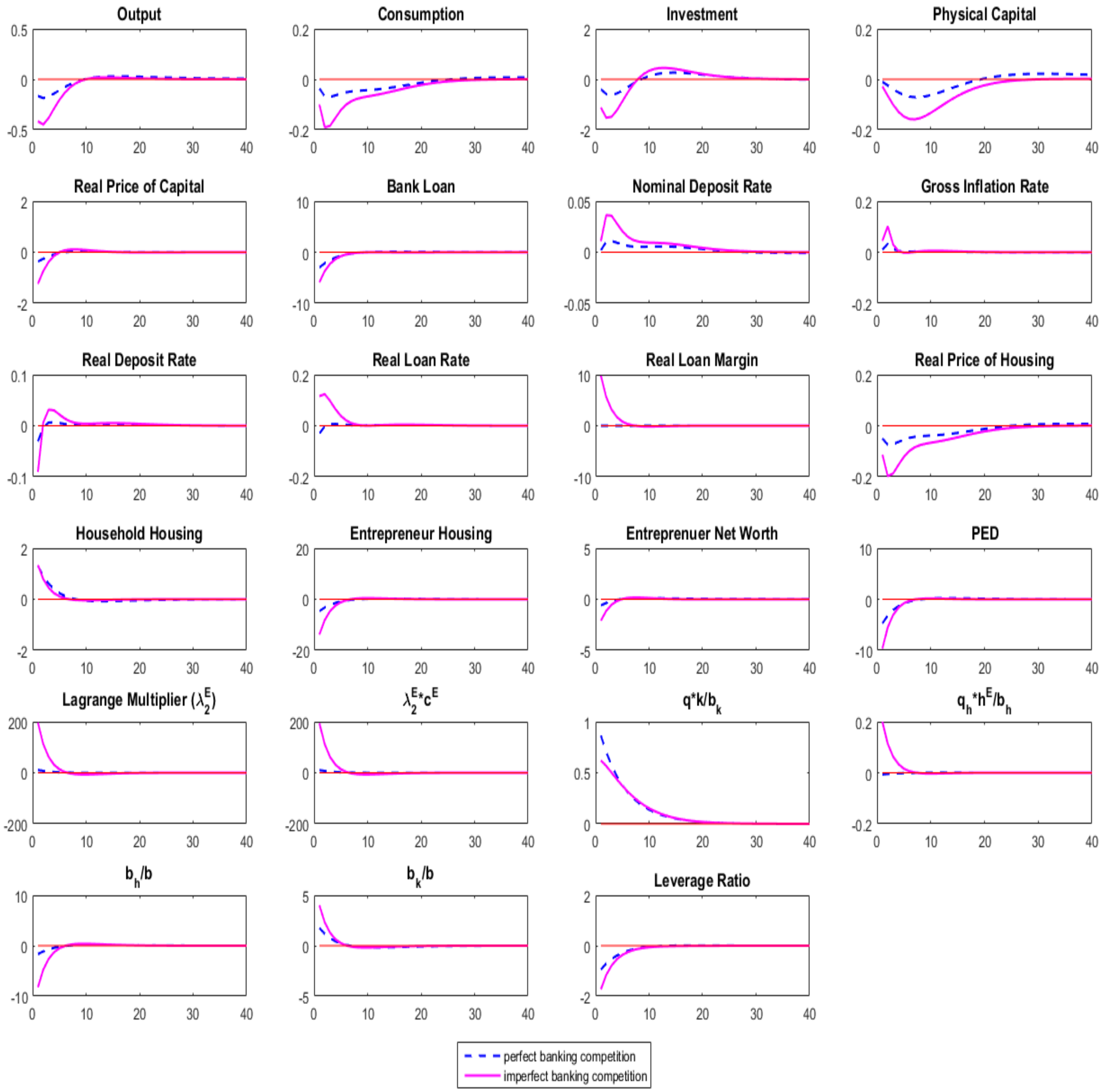

Figure 4: Impulse responses to a negative m_k shock

Note: Horizontal axis shows quarters after the shock that occurs at the beginning of period 1. Vertical axis shows the percentage deviation from the steady state. The blue dashed line corresponds to perfect banking competition, and the pink solid line corresponds to imperfect banking competition.

5.3.2 Negative Shock to $m_{k,t}$

In response to a one standard deviation ($\sigma_{m_k} = 0.01$) or 1% negative $m_{k,t}$ shock at the beginning of period 1 ($\sigma_{m_{k,t}} = 0.01$ in $t = 1$), imperfect banking competition is still found to amplify the responses of output, investment and physical capital. However, there are a few differences to notice from the negative shock to $m_{h,t}$.

First, the real price of capital falls by around 1.5% on impact under imperfect banking competition, which is more than twice the magnitude of the fall after the negative $m_{h,t}$ shock. The large fall in the real price of capital together with falling real price of housing and increasing real loan rate make the borrowing constraint bind more tightly, thus increasing $\lambda^E_{2,t}$ by around 200%, more than twice the increase in $\lambda^E_{2,t}$ after a negative $m_{h,t}$ shock. Second, the value of physical capital relative to the borrowing secured by the physical capital $\frac{q_t k_t}{b_{k,t}}$ rises after the negative $m_{k,t}$ shock, in contrast to all the previous cases. This is because a decrease in $m_{k,t}$ directly reduces the maximum amount of borrowing against the physical capital collateral $b_{k,t}$ and thus it triggers a larger percentage fall in $b_{k,t}$ from its steady state compared to the percentage fall in $q_t k_t$ from its steady state and hence the ratio $\frac{q_t k_t}{b_{k,t}}$ rises by around 0.6% on impact under imperfect banking competition.

A much larger increase in $\lambda^E_{2,t}$ and a rise in $\frac{q_t k_t}{b_{k,t}}$ both act to reduce PED_t more compared to the negative $m_{h,t}$ shock. As shown in Figure 4, PED_t falls by around 10%, approximately twice the fall of its counterpart after the negative $m_{h,t}$ shock. Hence, the increase in the real loan margin is much larger after the negative $m_{k,t}$ shock due to a more inelastic loan demand, giving rise to a more noticeable amplification effect.

6 Conclusions

In the presence of a binding collateral constraint, imperfect banking competition tends to amplify the responses of output, investment and physical capital after a contractionary monetary shock and negative collateral shocks, but slightly attenuate the responses of those variables after a negative productivity shock. In all cases, the real loan margin is found to be countercyclical, which tends to amplify the aggregate fluctuations. However, after the negative productivity shock, the attenuation effect from the net worth channel is more dominant, and hence aggregate fluctuations are slightly attenuated under imperfect banking competition.

The countercyclical real loan margin arises from a joint effect between banks' market power and the time-varying loan demand elasticity facing the banks. In this paper, the changing tightness of the binding borrowing constraint is one of the important factors that causes the loan demand elasticity to change over time. The borrowing constraint binds more tightly during bad times, reducing the entrepreneur's borrowing capacity and resulting in a more inelastic loan demand. Banks with the market power can take advantage of this lower loan demand elasticity, which then leads to a rise in the real loan rate. For a given real deposit rate, the real loan interest margin rises immediately after a negative shock. After the contractionary monetary shock and the negative collateral shocks, the countercyclical real loan margin has a clear amplification effect on the response of output.

However, the amplification effect of imperfect banking competition is absent after the negative productivity shock for two reasons. First, the magnitude of the countercyclical real loan margin is much smaller after the negative productivity shock compared to all the other cases. This is likely because the fall in the real loan rate after the negative productivity shock works to reduce the tightness of the binding borrowing constraint, unlike in all the other cases where the falling asset prices and the rising real loan rate both act to raise the tightness of the borrowing constraint. Second, the entrepreneur's net worth is attenuated under imperfect banking competition after the negative productivity shock effect, which tends to attenuate the response of output. Given that the magnitude of the countercyclical real loan margin is small and the net worth effect works in the opposite direction to the amplification effect from the countercyclical real loan margin, it is likely that the attenuation effect from the net worth channel is more dominant after the negative productivity shock.

Appendices

A Calvo Pricing

A.1 Optimal Pricing Equation

Substitute in $y^*_{t+s}(j)$ and rearrange:

$$Max_{p^*_t(j)} \quad \sum_{s=0}^{\infty} \theta^s E_t \left[\Lambda_{t,t+s} \left(\frac{p^*_t(j)}{p_{t+s}} - \frac{1}{x_{t+s}} \right) \left(\frac{p^*_t(j)}{p_{t+s}} \right)^{-\epsilon} y_{t+s} \right] \tag{58}$$

Take the first order condition:

$$\sum_{s=0}^{\infty} \theta^s E_t \Lambda_{t,t+s} \left[\left(\frac{1}{p_{t+s}} \right) \left(\frac{p^*_t(j)}{p_{t+s}} \right)^{-\epsilon} y_{t+s} + \left(\frac{p^*_t(j)}{p_{t+s}} - \frac{1}{x_{t+s}} \right) (-\epsilon) \left(\frac{p^*_t(j)}{p_{t+s}} \right)^{-\epsilon-1} \frac{y_{t+s}}{p_{t+s}} \right] = 0 \tag{59}$$

Simplify the above equation:

$$\sum_{s=0}^{\infty} \theta^s E_t \Lambda_{t,t+s} \left[(1-\epsilon) \left(\frac{y_{t+s}}{p_{t+s}} \right) \left(\frac{p^*_t(j)}{p_{t+s}} \right)^{-\epsilon} + \epsilon \frac{1}{x_{t+s}} p^*_t(j)^{-\epsilon-1} \left(\frac{1}{p_{t+s}} \right)^{-\epsilon} y_{t+s} \right] = 0 \tag{60}$$

Multiply by $\frac{p^*_t(j)^{\epsilon+1}}{1-\epsilon}$:

$$\sum_{s=0}^{\infty} \theta^s E_t \Lambda_{t,t+s} \left[p^*_t(j) \left(\frac{1}{p_{t+s}} \right)^{1-\epsilon} y_{t+s} + \frac{\epsilon}{1-\epsilon} \frac{1}{x_{t+s}} \left(\frac{1}{p_{t+s}} \right)^{-\epsilon} y_{t+s} \right] = 0 \tag{61}$$

Rearrange to solve for $p^*_t(j)$:

$$p^*_t(j) = \frac{\epsilon}{\epsilon-1} \frac{\sum_{s=0}^{\infty} \theta^s E_t \Lambda_{t,t+s} x^{-1}_{t+s} p^{\epsilon}_{t+s} y_{t+s}}{\sum_{s=0}^{\infty} \theta^s E_t \Lambda_{t,t+s} p^{\epsilon-1}_{t+s} y_{t+s}} = \frac{\epsilon}{\epsilon-1} \frac{\sum_{s=0}^{\infty} (\beta\theta)^s E_t u'(c_{t+s}) x^{-1}_{t+s} p^{\epsilon}_{t+s} y_{t+s}}{\sum_{s=0}^{\infty} (\beta\theta)^s E_t u'(c_{t+s}) p^{\epsilon-1}_{t+s} y_{t+s}} \tag{62}$$

To numerically implement the optimal pricing equation in Dynare, summarize the equation above with 2 recursive formulations such that:

$$p^*_t = \frac{\epsilon}{\epsilon-1} \frac{g_{1,t}}{g_{2,t}} \tag{63}$$

where

$$g_{1,t} = u'(c_t) p^{\epsilon}_t y_t x^{-1}_t + \beta\theta E_t(g_{1,t+1}) = \frac{1}{c_t} p^{\epsilon}_t y_t x^{-1}_t + \beta\theta E_t(g_{1,t+1}) \tag{64}$$

$$g_{2,t} = u'(c_t)p_t^{\epsilon-1}y_t + \beta\theta E_t(g_{2,t+1}) = \frac{1}{c_t}p_t^{\epsilon-1}y_t + \beta\theta E_t(g_{2,t+1}) \tag{65}$$

Let $f_{1,t} = p_t^{-\epsilon}g_{1,t}$, then

$$f_{1,t} = p_t^{-\epsilon}g_{1,t} = \frac{1}{c_t}y_t x_t^{-1} + \beta\theta E_t(\pi_{t+1}^{\epsilon}f_{1,t+1}) \tag{66}$$

Let $f_{2,t} = p_t^{1-\epsilon}g_{2,t}$, then

$$f_{2,t} = p_t^{1-\epsilon}g_{2,t} = \frac{1}{c_t}y_t + \beta\theta E_t(\pi_{t+1}^{\epsilon-1}f_{2,t+1}) \tag{67}$$

The optimal pricing equation $p_t^* = \frac{\epsilon}{\epsilon-1}\frac{g_{1,t}}{g_{2,t}}$ becomes:

$$p_t^* = \frac{\epsilon}{\epsilon-1}\frac{f_{1,t}p_t^{\epsilon}}{f_{2,t}p_t^{\epsilon-1}} = \frac{\epsilon}{\epsilon-1}\frac{f_{1,t}}{f_{2,t}}p_t \tag{68}$$

Divide both sides by p_{t-1} and let $\pi_t^* = \frac{p_t^*}{p_{t-1}}$ denote the gross reset price inflation rate to eliminate the price levels:

$$\pi_t^* = \frac{p_t^*}{p_{t-1}} = \frac{\epsilon}{\epsilon-1}\frac{f_{1,t}}{f_{2,t}}\pi_t \tag{69}$$

A.2 Aggregate Price Evolution

Rearrange the aggregate price index (30):

$$p_t^{1-\epsilon} = \int_0^1 p_t(j)^{1-\epsilon}dj \tag{70}$$

Following Sims (2014)[18], the above integral can be broken up into two parts by ordering the retailers along the unit interval:

$$p_t^{1-\epsilon} = \int_0^{1-\theta}(p_t^*)^{1-\epsilon}dj + \int_{1-\theta}^1 p_{t-1}(j)^{1-\epsilon}dj = (1-\theta)(p_t^*)^{1-\epsilon} + \int_{1-\theta}^1 p_{t-1}(j)^{1-\epsilon}dj \tag{71}$$

Given the assumptions that the price-adjusting retailers in each period are randomly chosen and the number of retailers is large, the integral of individual prices over $[1-\theta, 1]$ of the unit interval is equal to a proportion θ of the integral over the entire unit interval, where θ is the length of the subset $[1-\theta, 1]$. That is,

$$\int_{1-\theta}^1 p_{t-1}(j)^{1-\epsilon}dj = \theta\int_0^1 p_{t-1}(j)^{1-\epsilon}dj = \theta p_{t-1}^{1-\epsilon} \tag{72}$$

[18] https://www3.nd.edu/~esims1/new_keynesian_2014.pdf

Hence,

$$p_t^{1-\epsilon} = (1-\theta)(p_t^*)^{1-\epsilon} + \theta p_{t-1}^{1-\epsilon} \tag{73}$$

To compute the model numerically, it is necessary to rewrite the price evolution in terms of the inflation rates because the price level may not be stationary. Eliminating the price levels in the equation above by dividing both sides by $p_{t-1}^{1-\epsilon}$:

$$\left(\frac{p_t}{p_{t-1}}\right)^{1-\epsilon} = \theta + (1-\theta)\left(\frac{p_t^*}{p_{t-1}}\right)^{1-\epsilon} \tag{74}$$

Let $\pi_t \equiv \frac{p_t}{p_{t-1}}$ and $\pi_t^* \equiv \frac{p_t^*}{p_{t-1}}$ denote the gross inflation rate and the gross reset price inflation rate respectively, the equation above can be rewritten as:

$$\pi_t^{1-\epsilon} = \theta + (1-\theta)(\pi_t^*)^{1-\epsilon} \tag{75}$$

A.3 Price Dispersion

Use the Calvo assumption to break up the integral into two parts by ordering the retailers along the unit interval:

$$f_{3,t} \equiv \int_0^1 \left[\frac{p_t(j)}{p_t}\right]^{-\epsilon} dj = \int_0^{1-\theta} \left(\frac{p_t^*}{p_t}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left[\frac{p_{t-1}(j)}{p_t}\right]^{-\epsilon} dj \tag{76}$$

Rearrange and simplify by using the definitions for π_t and π_t^*:

$$f_{3,t} = \int_0^{1-\theta} \left(\frac{p_t^*}{p_{t-1}}\frac{p_{t-1}}{p_t}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left[\frac{p_{t-1}(j)}{p_{t-1}}\frac{p_{t-1}}{p_t}\right]^{-\epsilon} dj = (1-\theta)(\pi_t^*)^{-\epsilon}\pi_t^\epsilon + \pi_t^\epsilon \int_{1-\theta}^1 \left[\frac{p_{t-1}(j)}{p_{t-1}}\right]^{-\epsilon} dj \tag{77}$$

Use the same method as in Appendix A.2 to simplify the last term in the equation above:

$$\int_{1-\theta}^1 \left[\frac{p_{t-1}(j)}{p_{t-1}}\right]^{-\epsilon} dj = \theta \int_0^1 \left[\frac{p_{t-1}(j)}{p_{t-1}}\right]^{-\epsilon} dj = \theta f_{3,t-1} \tag{78}$$

Hence, the price dispersion $f_{3,t}$ can be written recursively:

$$f_{3,t} = (1-\theta)(\pi_t^*)^{-\epsilon}\pi_t^\epsilon + \pi_t^\epsilon \theta f_{3,t-1} \tag{79}$$

As can be seen, the index j has been eliminated in the above expression. Consequently, there is no need to keep track of the individual prices.

The final consumption good output y_t is:

$$y_t = \frac{y_{w,t}}{f_{3,t}} = \frac{y_{w,t}}{(1-\theta)(\pi_t^*)^{-\epsilon}\pi_t^\epsilon + \pi_t^\epsilon \theta f_{3,t-1}} \tag{80}$$

The real profit Π_t^R made by the continuum of unit mass retailers is:

$$\Pi_t^R = \int_0^1 \left[\frac{p_t(j)}{p_t} y_t(j) - \frac{1}{x_t} y_t(j)\right] dj = \int_0^1 \frac{p_t(j)}{p_t} y_t(j) dj - \frac{1}{x_t}\int_0^1 y_t(j) dj \tag{81}$$

Use retailer $j's$ individual demand function $y_t(j) = \left[\frac{p_t(j)}{p_t}\right]^{-\epsilon} y_t$, the wholesale good output expression $y_{w,t} = \int_0^1 y_t(j)dj$, and the aggregate price index $p_t = \left[\int_0^1 p_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$ to simplify:

$$\Pi_t^R = \int_0^1 \frac{p_t(j)}{p_t}\left[\frac{p_t(j)}{p_t}\right]^{-\epsilon} y_t dj - \frac{y_{w,t}}{x_t} = y_t p_t^{\epsilon-1}\int_0^1 p_t(j)^{1-\epsilon} dj - \frac{y_{w,t}}{x_t} = y_t - \frac{y_{w,t}}{x_t} = (\frac{1}{f_{3,t}} - \frac{1}{x_t}) y_{w,t} \tag{82}$$

B Solving the Entrepreneur's Problem

The proof resembles the approach used by Andrés and Arce (2012) in solving for c_t^E and b_t. Substitute $\lambda_{1,t}^E = \frac{1}{c_t^E}$ and $\lambda_{2,t}^E = \frac{1}{c_t^E} - \beta^E E_t(\frac{1}{c_{t+1}^E}\frac{R_{b,t}}{\pi_{t+1}})$ into the first order condition with respect to k_t and rearrange:

$$\frac{q_t - m_{k,t}E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}}\right]}{c_t^E} = \beta^E E_t \frac{1}{c_{t+1}^E}\left\{\frac{\alpha y_{w,t+1}}{x_{t+1}k_t} + q_{t+1}(1-\delta) - \frac{m_{k,t}}{\pi_{t+1}}E_t[q_{t+1}(1-\delta)\pi_{t+1}]\right\} \tag{83}$$

Multiply both sides by k_t:

$$\frac{q_t k_t - m_{k,t}E_t\left[\frac{q_{t+1}k_t(1-\delta)\pi_{t+1}}{R_{b,t}}\right]}{c_t^E} = \beta^E E_t \frac{1}{c_{t+1}^E}\left\{\frac{\alpha y_{w,t+1}}{x_{t+1}} + q_{t+1}k_t(1-\delta) - \frac{m_{k,t}}{\pi_{t+1}}E_t[q_{t+1}k_t(1-\delta)\pi_{t+1}]\right\} \tag{84}$$

Similarly, substitute the expressions for $\lambda_{1,t}^E$ and $\lambda_{2,t}^E$ into the first order condition with respect to h_t^E and rearrange:

$$\frac{q_{h,t} - m_{h,t}E_t\left(\frac{q_{h,t+1}\pi_{t+1}}{R_{b,t}}\right)}{c_t^E} = \beta^E E_t \frac{1}{c_{t+1}^E}\left[\frac{\upsilon y_{w,t+1}}{x_{t+1}h_t^E} + q_{h,t+1} - \frac{m_{h,t}}{\pi_{t+1}}E_t(q_{h,t+1}\pi_{t+1})\right] \tag{85}$$

Multiply both sides by h_t^E:

$$\frac{q_{h,t}h_t^E - m_{h,t}E_t\left(\frac{q_{h,t+1}h_t^E\pi_{t+1}}{R_{b,t}}\right)}{c_t^E} = \beta^E E_t \frac{1}{c_{t+1}^E}\left[\frac{v y_{w,t+1}}{x_{t+1}} + q_{h,t+1}h_t^E - \frac{m_{h,t}}{\pi_{t+1}}E_t(q_{h,t+1}h_t^E\pi_{t+1})\right] \tag{86}$$

Adding up equation (84) and (86) and substitute in the binding borrowing constraint (48) to simplify:

$$\frac{q_t k_t + q_{h,t}h_t^E - b_t}{c_t^E} = \beta^E E_t \frac{1}{c_{t+1}^E}\left[\frac{(\alpha+v)y_{w,t+1}}{x_{t+1}} + q_{t+1}k_t(1-\delta) + q_{h,t+1}h_t^E - \frac{R_{b,t}b_t}{\pi_{t+1}}\right] \tag{87}$$

Given the definition for the entrepreneur's net worth nw_t shown in (21):

$$nw_t \equiv \frac{(\alpha+v)y_{w,t}}{x_t} + q_t(1-\delta)k_{t-1} + q_{h,t}h_{t-1}^E - \frac{R_{b,t-1}b_{t-1}}{\pi_t} \tag{88}$$

Rewrite the flow of funds constraint above in terms of nw_t:

$$c_t^E + q_t k_t + q_{h,t}h_t^E = nw_t + b_t \tag{89}$$

Using (21) and (89), equation (87) can be written as:

$$\frac{nw_t - c_t^E}{c_t^E} = \beta^E E_t\left(\frac{nw_{t+1}}{c_{t+1}^E}\right) \tag{90}$$

Conjecture that $c_t^E = \gamma nw_t$ and substitute the conjecture into the equation above:

$$\frac{(1-\gamma)nw_t}{nw_t} = \beta^E E_t\left[\frac{nw_{t+1}}{\gamma nw_{t+1}}\right] \tag{91}$$

Hence, $\gamma = (1-\beta^E)$, $c_t^E = (1-\beta^E)nw_t$ and $b_t = q_t k_t + q_{h,t}h_t^E - \beta^E nw_t$.

C Market Loan Demand Function

Rewrite the first order condition with respect to k_t (83) as:

$$q_t - m_{k,t}E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}}\right] = \beta^E E_t \frac{c_t^E}{c_{t+1}^E}\left[\frac{\alpha z_{t+1}(l_{t+1}^E)^{1-\alpha-v}}{x_{t+1}}\right]k_t^{\alpha-1}(h_t^E)^v$$
$$+ \beta^E E_t \frac{c_t^E}{c_{t+1}^E}\left\{q_{t+1}(1-\delta) - \frac{m_{k,t}}{\pi_{t+1}}E_t[q_{t+1}(1-\delta)\pi_{t+1}]\right\} \tag{92}$$

Use notations $A_{k,t}$, $B_{k,t}$ and $C_{k,t}$ to simplify the above expression:

$$A_{k,t} = q_t - m_{k,t} E_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}} \right] \tag{93}$$

$$B_{k,t} = \beta^E E_t \frac{c_t^E}{c_{t+1}^E} \left[\frac{\alpha z_{t+1} (l_{t+1}^E)^{1-\alpha-v}}{x_{t+1}} \right] \tag{94}$$

$$C_{k,t} = \beta^E E_t \frac{c_t^E}{c_{t+1}^E} \left\{ q_{t+1}(1-\delta) - \frac{m_{k,t}}{\pi_{t+1}} E_t[q_{t+1}(1-\delta)\pi_{t+1}] \right\} \tag{95}$$

Hence, rearrange (92) to solve for k_t:

$$k_t = \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{\frac{1}{\alpha-1}} (h_t^E)^{\frac{v}{1-\alpha}} \tag{96}$$

Similarly, rewrite the first order condition with respect to h_t^E (85) using the following notations:

$$A_{h,t} = q_{h,t} - m_{h,t} E_t \left(\frac{q_{h,t+1}\pi_{t+1}}{R_{b,t}} \right) \tag{97}$$

$$B_{h,t} = \beta^E E_t \frac{c_t^E}{c_{t+1}^E} \left[\frac{v z_{t+1} (l_{t+1}^E)^{1-\alpha-v}}{x_{t+1}} \right] \tag{98}$$

$$C_{h,t} = \beta^E E_t \frac{c_t^E}{c_{t+1}^E} \left[q_{h,t+1} - \frac{m_{h,t}}{\pi_{t+1}} E_t(q_{h,t+1}\pi_{t+1}) \right] \tag{99}$$

Hence, (85) can be written as:

$$A_{h,t} = B_{h,t} k_t^{\alpha} (h_t^E)^{v-1} + C_{h,t} \tag{100}$$

Substitute (96) into the above equation:

$$A_{h,t} = B_{h,t} \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{\frac{\alpha}{\alpha-1}} (h_t^E)^{\frac{\alpha v}{1-\alpha}} (h_t^E)^{v-1} + C_{h,t} \tag{101}$$

Rearrange the equation above to solve for h_t^E:

$$h_t^E = \left(\frac{A_{h,t} - C_{h,t}}{B_{h,t}} \right)^{\frac{1-\alpha}{v-1+\alpha}} \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{\frac{\alpha}{v-1+\alpha}} = \left(\frac{A_{h,t} - C_{h,t}}{B_{h,t}} \right)^{u_1} \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{u_2} \tag{102}$$

where

$$u_1 \equiv \frac{1-\alpha}{v-1+\alpha} \tag{103}$$

$$u_2 \equiv \frac{\alpha}{v-1+\alpha} \tag{104}$$

Substitute the expression for h_t^E (102) into (96):

$$k_t = \left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{\frac{1}{\alpha-1}} \left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{\frac{v}{v-1+\alpha}} \left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{\frac{\alpha v}{(v-1+\alpha)(1-\alpha)}} = \left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_3} \left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{u_4} \tag{105}$$

where

$$u_3 \equiv \frac{1-v}{v-1+\alpha} \tag{106}$$

$$u_4 \equiv \frac{v}{v-1+\alpha} \tag{107}$$

Note that $R_{b,t}$ is present in both $A_{h,t}$ and $A_{k,t}$. Differentiate the two choice variables h_t^E and k_t with respect to $R_{b,t}$:

$$\frac{\partial h_t^E}{\partial R_{b,t}} = u_1 \left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{u_1-1} \left(\frac{D_{h,t}}{B_{h,t}}\right) \left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_2} + \left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{u_1} u_2 \left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_2-1} \left(\frac{D_{k,t}}{B_{k,t}}\right) \tag{108}$$

$$\frac{\partial k_t}{\partial R_{b,t}} = u_4 \left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{u_4-1} \left(\frac{D_{h,t}}{B_{h,t}}\right) \left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_3} + \left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{u_4} u_3 \left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_3-1} \left(\frac{D_{k,t}}{B_{k,t}}\right) \tag{109}$$

where

$$D_{h,t} = m_{h,t} E_t \left(\frac{q_{h,t+1}\pi_{t+1}}{R_{b,t}^2}\right) \tag{110}$$

$$D_{k,t} = m_{k,t} E_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}^2}\right] \tag{111}$$

This method of solving for $\frac{\partial h_t^E}{\partial R_{b,t}}$ and $\frac{\partial k_t}{\partial R_{b,t}}$ is the same as doing implicit differentiation in the two first order conditions, (83) and (85).

D Elasticity of Loan Demand

$$\begin{aligned}
PED_t &= -\frac{NR_{b,t}}{b_t}\frac{\partial b_t}{\partial R_{b,t}} \\
&= -\frac{NR_{b,t}}{b_t}\left\{-\frac{b_t}{R_{b,t}} + m_{h,t}E_t\left[\frac{q_{h,t+1}\pi_{t+1}}{R_{b,t}}\right]\frac{\partial h_t^E}{\partial R_{b,t}} + m_{k,t}E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}}\right]\frac{\partial k_t}{\partial R_{b,t}}\right\} \\
&= N - \frac{NR_{b,t}}{b_t}m_{h,t}E_t\left[\frac{q_{h,t+1}\pi_{t+1}}{R_{b,t}}\right]\frac{\partial h_t^E}{\partial R_{b,t}} - \frac{NR_{b,t}}{b_t}m_{k,t}E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}}\right]\frac{\partial k_t}{\partial R_{b,t}} \\
&= N - \frac{NR_{b,t}}{b_t}\frac{b_{h,t}}{h_t^E}\frac{\partial h_t^E}{\partial R_{b,t}} - \frac{NR_{b,t}}{b_t}\frac{b_{k,t}}{k_t}\frac{\partial k_t}{\partial R_{b,t}} \\
&= N\left(1 - \frac{b_{h,t}}{b_t}\frac{\partial h_t^E}{\partial R_{b,t}}\frac{R_{b,t}}{h_t^E} - \frac{b_{k,t}}{b_t}\frac{\partial k_t}{\partial R_{b,t}}\frac{R_{b,t}}{k_t}\right)
\end{aligned} \tag{112}$$

where $b_{h,t} = m_{h,t}E_t\left[\frac{q_{h,t+1}h_t^E\pi_{t+1}}{R_{b,t}}\right]$ and $b_{k,t} = m_{k,t}E_t\left[\frac{q_{t+1}k_t(1-\delta)\pi_{t+1}}{R_{b,t}}\right]$. To find $\frac{\partial h_t^E}{\partial R_{b,t}}\frac{R_{b,t}}{h_t^E}$ and $\frac{\partial k_t}{\partial R_{b,t}}\frac{R_{b,t}}{k_t}$, use (108) and substitute in the expressions for $D_{h,t}$ (110) and $D_{k,t}$ (111):

$$\begin{aligned}
\frac{\partial h_t^E}{\partial R_{b,t}}\frac{R_{b,t}}{h_t^E} =& \frac{R_{b,t}}{h_t^E}u_1\left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{u_1-1}\left(\frac{m_{h,t}E_t\left(q_{h,t+1}\pi_{t+1}R_{b,t}^{-2}\right)}{B_{h,t}}\right)\left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_2} \\
&+ \frac{R_{b,t}}{h_t^E}\left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{u_1}u_2\left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_2-1}\left(\frac{m_{k,t}E_t\left(q_{t+1}(1-\delta)\pi_{t+1}R_{b,t}^{-2}\right)}{B_{k,t}}\right)
\end{aligned} \tag{113}$$

Since $\frac{1}{h_t^E} = \left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{-u_1}\left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{-u_2}$ and let $\Lambda_{t,t+1}^E \equiv \beta^E\frac{u'(c_{t+1}^E)}{u'(c_t^E)} = \beta^E\frac{c_t^E}{c_{t+1}^E}$, the expression above can be simplified to:

$$\begin{aligned}
\frac{\partial h_t^E}{\partial R_{b,t}}\frac{R_{b,t}}{h_t^E} &= u_1\left(\frac{A_{h,t}-C_{h,t}}{B_{h,t}}\right)^{-1}\left(\frac{m_{h,t}E_t\left(q_{h,t+1}\pi_{t+1}R_{b,t}^{-1}\right)}{B_{h,t}}\right) \\
&\quad + u_2\left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{-1}\left(\frac{m_{k,t}E_t\left(q_{t+1}(1-\delta)\pi_{t+1}R_{b,t}^{-1}\right)}{B_{k,t}}\right) \\
&= \frac{u_1 m_{h,t}E_t\left(q_{h,t+1}\pi_{t+1}R_{b,t}^{-1}\right)}{q_{h,t} - m_{h,t}E_t\left(q_{h,t+1}\pi_{t+1}R_{b,t}^{-1}\right) - E_t\left(\Lambda_{t,t+1}^E q_{h,t+1}(1-m_{h,t})\right)} \\
&\quad + \frac{u_2 m_{k,t}E_t\left(q_{t+1}(1-\delta)\pi_{t+1}R_{b,t}^{-1}\right)}{q_t - m_{k,t}E_t\left(q_{t+1}(1-\delta)\pi_{t+1}R_{b,t}^{-1}\right) - E_t\left(\Lambda_{t,t+1}^E q_{t+1}(1-\delta)(1-m_{k,t})\right)}
\end{aligned} \tag{114}$$

Since $m_{h,t}E_t\left(q_{h,t+1}\pi_{t+1}R_{b,t}^{-1}\right)=\frac{b_{h,t}}{h_t^E}$ and $m_{k,t}E_t\left(q_{t+1}(1-\delta)\pi_{t+1}R_{b,t}^{-1}\right)=\frac{b_{k,t}}{k_t}$, further simplify the above expression to:

$$\begin{aligned}\frac{\partial h_t^E}{\partial R_{b,t}}\frac{R_{b,t}}{h_t^E}&=\frac{u_1}{\frac{q_{h,t}h_t^E}{b_{h,t}}-E_t\left(\Lambda_{t,t+1}^E q_{h,t+1}(1-m_{h,t})h_t^E b_{h,t}^{-1}\right)-1}\\&+\frac{u_2}{\frac{q_t k_t}{b_{k,t}}-E_t\left(\Lambda_{t,t+1}^E q_{t+1}(1-m_{k,t})(1-\delta)k_t b_{k,t}^{-1}\right)-1}\\&=\frac{u_1}{\frac{q_{h,t}h_t^E}{b_{h,t}}-\frac{1-m_{h,t}}{m_{h,t}}E_t\left(\Lambda_{t,t+1}^E\frac{R_{b,t}}{\pi_{t+1}}\right)-1}+\frac{u_2}{\frac{q_t k_t}{b_{k,t}}-\frac{1-m_{k,t}}{m_{k,t}}E_t\left(\Lambda_{t,t+1}^E\frac{R_{b,t}}{\pi_{t+1}}\right)-1}\end{aligned}\tag{115}$$

This can be further simplified to:

$$\frac{\partial h_t^E}{\partial R_{b,t}}\frac{R_{b,t}}{h_t^E}=\frac{u_1}{\frac{q_{h,t}h_t^E}{b_{h,t}}-\frac{1-m_{h,t}}{m_{h,t}}E_t\left(\Lambda_{t,t+1}^E\frac{R_{b,t}}{\pi_{t+1}}\right)-1}+\frac{u_2}{\frac{q_t k_t}{b_{k,t}}-\frac{1-m_{k,t}}{m_{k,t}}E_t\left(\Lambda_{t,t+1}^E\frac{R_{b,t}}{\pi_{t+1}}\right)-1}\tag{116}$$

To see the role of the Lagrange multiplier $\lambda_{2,t}^E$, use (15) and (16) to write $E_t\left(\Lambda_{t,t+1}^E\frac{R_{b,t}}{\pi_{t+1}}\right)$ as $(1-\lambda_{2,t}^E c_t^E)$ so that

$$\frac{\partial h_t^E}{\partial R_{b,t}}\frac{R_{b,t}}{h_t^E}=\frac{u_1}{\frac{q_{h,t}h_t^E}{b_{h,t}}+(\frac{1}{m_{h,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{h,t}}}+\frac{u_2}{\frac{q_t k_t}{b_{k,t}}+(\frac{1}{m_{k,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{k,t}}}\tag{117}$$

Similarly,

$$\frac{\partial k_t}{\partial R_{b,t}}\frac{R_{b,t}}{k_t}=\frac{u_4}{\frac{q_{h,t}h_t^E}{b_{h,t}}+(\frac{1}{m_{h,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{h,t}}}+\frac{u_3}{\frac{q_t k_t}{b_{k,t}}+(\frac{1}{m_{k,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{k,t}}}\tag{118}$$

Substituting (117) and (118) into (112) gives:

$$\begin{aligned}PED_t&=N\left(1-\frac{\frac{b_{h,t}}{b_t}u_1+\frac{b_{k,t}}{b_t}u_4}{\frac{q_{h,t}h_t^E}{b_{h,t}}+(\frac{1}{m_{h,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{h,t}}}-\frac{\frac{b_{h,t}}{b_t}u_2+\frac{b_{k,t}}{b_t}u_3}{\frac{q_t k_t}{b_{k,t}}+(\frac{1}{m_{k,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{k,t}}}\right)\\&=N\left(1+\frac{\frac{b_{h,t}}{b_t}\frac{1-\alpha}{1-v-\alpha}+\frac{b_{k,t}}{b_t}\frac{v}{1-v-\alpha}}{\frac{q_{h,t}h_t^E}{b_{h,t}}+(\frac{1}{m_{h,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{h,t}}}+\frac{\frac{b_{h,t}}{b_t}\frac{\alpha}{1-v-\alpha}+\frac{b_{k,t}}{b_t}\frac{1-v}{1-v-\alpha}}{\frac{q_t k_t}{b_{k,t}}+(\frac{1}{m_{k,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{k,t}}}\right)\\&=N\left(1+\frac{\frac{b_{h,t}}{b_t}+\frac{v}{1-v-\alpha}}{\frac{q_{h,t}h_t^E}{b_{h,t}}+(\frac{1}{m_{h,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{h,t}}}+\frac{\frac{\alpha}{1-v-\alpha}+\frac{b_{k,t}}{b_t}}{\frac{q_t k_t}{b_{k,t}}+(\frac{1}{m_{k,t}}-1)\lambda_{2,t}^E c_t^E-\frac{1}{m_{k,t}}}\right)\end{aligned}\tag{119}$$

E Dynare Model Block

For clarity of the representation, the equations here are presented in their original forms, without doing the log transformation of the variables. When they are used in Dynare, variables are written in an exponential form, i.e. exp(variable), to implement log-linearisation.

E.1 Households

1) Household intertemporal consumption Euler equation:

$$1 = \beta E_t[\frac{c_t}{c_{t+1}} \frac{R_t}{\pi_{t+1}}] \tag{120}$$

2) Household intratemporal consumption-labor choice:

$$\frac{\phi}{1 - l_t} = \frac{w_t}{c_t} \tag{121}$$

3) Household demand for real estate:

$$\frac{\phi_h}{h_t} + \beta E_t \left(\frac{1}{c_{t+1}} q_{h,t+1} \right) = \frac{1}{c_t} q_{h,t} \tag{122}$$

E.2 Entrepreneurs

1) Entrepreneur's utility maximization with respect to k_t ($l_t = l_t^E$ in equilibrium):

$$\frac{q_t}{c_t^E} = \beta^E E_t \left\{ \frac{1}{c_{t+1}^E} \left[\frac{\alpha y_{w,t+1}}{x_{t+1} k_t} + (1-\delta) q_{t+1} \right] \right\} + \left[\frac{1}{c_t^E} - \beta^E E_t \left(\frac{1}{c_{t+1}^E} \frac{R_{b,t}}{\pi_{t+1}} \right) \right] m_{k,t} E_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}} \right] \tag{123}$$

2) Entrepreneur's utility maximization with respect to l_t :

$$w_t = \frac{(1 - \alpha - v) y_{w,t}}{x_t l_t^E} \tag{124}$$

3) Entrepreneur's borrowing:

$$b_t = q_t k_t + q_{h,t} h_t^E - \beta^E n w_t \tag{125}$$

4) Binding Borrowing Constraint:

$$b_t = m_{h,t} E_t \left[\frac{q_{h,t+1} h_t^E \pi_{t+1}}{R_{b,t}} \right] + m_{k,t} E_t \left[\frac{q_{t+1} k_t (1-\delta) \pi_{t+1}}{R_{b,t}} \right] \tag{126}$$

5) Net investment in physical capital:

$$i_t = k_t - (1-\delta)k_{t-1} \tag{127}$$

6) AR(1) process for productivity shock:

$$lnz_t = \psi lnz_{t-1} + e_{zt} \tag{128}$$

7) AR(1) process for $m_{h,t}$ shock:

$$lnm_{h,t} = \psi_{m_h} lnm_{h,t-1} + e_{m_h,t} \tag{129}$$

8) AR(1) process for $m_{k,t}$ shock:

$$lnm_{k,t} = \psi_{m_k} lnm_{k,t-1} + e_{m_k,t} \tag{130}$$

9) Wholesale good output:

$$y_{w,t} = z_t k_{t-1}^{\alpha} (h_{t-1}^E)^v (l_t^E)^{1-\alpha-v} \tag{131}$$

10) Entrepreneur's consumption:

$$c_t^E = (1-\beta^E)nw_t \tag{132}$$

11) Entrepreneur's net worth:

$$nw_t = \frac{(\alpha+v)y_{w,t}}{x_t} + q_t(1-\delta)k_{t-1} + q_{h,t}h_{t-1}^E - \frac{R_{b,t-1}b_{t-1}}{\pi_t} \tag{133}$$

E.3 Retailers

1) Optimal price rule:

$$f_{1,t} = \frac{1}{c_t} y_t x_t^{-1} + \beta\theta E_t(\pi_{t+1}^{\epsilon} f_{1,t+1}) \tag{134}$$

2) Optimal price rule:

$$f_{2,t} = \frac{1}{c_t} y_t + \beta\theta E_t(\pi_{t+1}^{\epsilon-1} f_{2,t+1}) \tag{135}$$

3) Gross reset price inflation rate:

$$\pi_t^* = \frac{\epsilon}{\epsilon-1}\frac{f_{1,t}}{f_{2,t}}\pi_t \tag{136}$$

4) Aggregate price evolution (written in terms of gross inflation rates):

$$\pi_t^{1-\epsilon} = \theta + (1-\theta)(\pi_t^*)^{1-\epsilon} \tag{137}$$

5) Recursive form of price dispersion:

$$f_{3,t} = (1-\theta)(\pi_t^*)^{-\epsilon}\pi_t^{\epsilon} + \pi_t^{\epsilon}\theta f_{3,t-1} \tag{138}$$

E.4 Capital Producers

$$q_t = 1 + \frac{\chi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 + \chi\frac{i_t}{i_{t-1}}\left(\frac{i_t}{i_{t-1}} - 1\right) - \chi E_t\Lambda_{t,t+1}\left(\frac{i_{t+1}}{i_t}\right)^2\left(\frac{i_{t+1}}{i_t} - 1\right) \tag{139}$$

E.5 Banking Sector

1) Perfectly competitive banks' profit maximization with respect to b_t:

$$E_t\left[\Lambda_{t,t+1}\frac{1}{\pi_{t+1}}(R_{b,t} - R_t)\right] = 0 \tag{140}$$

2) Bank j's profit maximization with respect to $b_t(j)$ under Cournot competition:

$$E_t\left\{\Lambda_{t,t+1}\frac{1}{\pi_{t+1}}\left[\frac{\partial R_{b,t}}{\partial b_t}\frac{b_t}{N} + R_{b,t} - R_t\right]\right\} = 0 \tag{141}$$

where $\frac{\partial b_t}{\partial R_{b,t}}$ can be seen from (50).
3) Fisher equation ($R_{rb,t}$ is the real loan rate):

$$R_{rb,t} = \frac{R_{b,t}}{\pi_{t+1}} \tag{142}$$

4) Real loan margin (under imperfect banking competition only):

$$RLM_t = R_{rb,t} - R_{r,t} \tag{143}$$

E.6 Central Bank

1) Taylor rule:

$$R_t = (1-\rho_r)R + \rho_r R_{t-1} + (1-\rho_r)\left[\kappa_\pi(\pi_t - \pi) + \kappa_y ln\left(\frac{y_t}{y}\right)\right] + e_{r,t} \tag{144}$$

2) Fisher equation ($R_{r,t}$ is the real interest rate controlled by the central bank):

$$R_{r,t} = \frac{R_t}{\pi_{t+1}} \tag{145}$$

E.7 Market Clearing

1) Final output:

$$y_t = \frac{z_t k_{t-1}^{\alpha} (h_{t-1}^E)^v (l_t^E)^{1-\alpha-v}}{f_{3,t}} \tag{146}$$

2) Aggregate Resource Constraint:

$$c_t + i_t + \frac{\chi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t + c_t^E = y_t \tag{147}$$

3) Housing market clearing condition, with the total fixed housing supply normalised to 1:

$$h_t + h_t^E = 1 \tag{148}$$

F Log-linearisation

F.1 Household's Consumption-Leisure Choice

Let $\widehat{var_t}$ denote the log deviation of a variable var_t from its steady state value var, then $\widehat{var_t} = ln(var_t) - ln(var)$. Combine (21) and (48) to rewrite the expression for nw_t as:

$$nw_t = \frac{(\alpha + v) y_{w,t}}{x_t} + (1 - m_{k,t}) q_t (1-\delta) k_{t-1} + (1 - m_{h,t}) q_{h,t} h_{t-1}^E \tag{149}$$

By definition of $\widehat{var_t}$, $var_t = (var)exp(\widehat{var_t})$, hence, rewrite the above expression as:

$$\begin{aligned} nwexp(\widehat{nw_t}) = & \frac{(\alpha + v) y_w exp(\widehat{y_{w,t}} - \widehat{x_t})}{x} + (1 - m_k exp(\widehat{m_{k,t}})) qk(1-\delta) exp(\widehat{q_t} + \widehat{k_{t-1}}) \\ & + (1 - m_h exp(\widehat{m_{h,t}})) q_h h^E exp(\widehat{q_{h,t}} + \widehat{h_{t-1}^E}) \end{aligned} \tag{150}$$

Replace $exp(\widehat{var_t})$ with its approximation $exp(\widehat{var_t}) \approx 1 + \widehat{var_t}$ and eliminate the constant terms using the steady state relationship to get:

$$\begin{aligned} nw\widehat{nw_t} = & \frac{(\alpha + v) y_w (\widehat{y_{w,t}} - \widehat{x_t})}{x} + (1 - m_k) qk(1-\delta)(\widehat{q_t} + \widehat{k_{t-1}}) + (1 - m_h) q_h h^E (\widehat{q_{h,t}} + \widehat{h_{t-1}^E}) \\ & - m_k qk(1-\delta)\widehat{m_{k,t}} - m_h q_h h^E \widehat{m_{h,t}} \end{aligned} \tag{151}$$

According to (24), in the steady state

$$nw = \frac{1}{\beta^E}(qk + q_h h^E - b) \tag{152}$$

Hence,

$$\begin{aligned}
\widehat{nw_t} =& \frac{\beta^E}{qk + q_h h^E - b}\Bigg[\frac{(\alpha+v)y_w(\widehat{y_{w,t}} - \widehat{x}_t)}{x} + (1-m_k)qk(1-\delta)(\widehat{q}_t + \widehat{k_{t-1}}) \\
&+ (1-m_h)q_h h^E(\widehat{q_{h,t}} + \widehat{h^E_{t-1}}) - m_k qk(1-\delta)\widehat{m_{k,t}} - m_h q_h h^E \widehat{m_{h,t}}\Bigg] \\
=& \frac{\beta^E}{\frac{qk+q_h h^E}{b} - 1}\Bigg[\frac{(\alpha+v)y_w}{xb}(\widehat{y_{w,t}} - \widehat{x}_t) + (1-m_k)\frac{qk}{b}(1-\delta)(\widehat{q}_t + \widehat{k_{t-1}}) \\
&+ (1-m_h)\frac{q_h h^E}{b}(\widehat{q_{h,t}} + \widehat{h^E_{t-1}}) - m_k\frac{qk}{b}(1-\delta)\widehat{m_{k,t}} - m_h\frac{q_h h^E}{b}\widehat{m_{h,t}}\Bigg] \\
=& \frac{\beta^E}{\frac{qk+q_h h^E}{b} - 1}\Bigg\{\frac{(\alpha+v)y_w}{xb}(\widehat{y_{w,t}} - \widehat{x}_t) + \frac{qk}{b}(1-\delta)[(1-m_k)(\widehat{q}_t + \widehat{k_{t-1}}) - m_k\widehat{m_{k,t}}] \\
&+ \frac{q_h h^E}{b}[(1-m_h)(\widehat{q_{h,t}} + \widehat{h^E_{t-1}}) - m_h\widehat{m_{h,t}}]\Bigg\}
\end{aligned} \tag{153}$$

G Model Parameters and Steady State Values

Table 1: Calibrated Parameters

Parameter	Value	Value
Households		
β	0.995	Subjective discount factor
ϕ	1.45	Relative utility weight on leisure time
ϕ_h	0.1	Relative utility weight on housing
Entrepreneurs		
α	0.33	Physical capital share
δ	0.025	Depreciation rate
β^E	0.97	Subjective discount factor
υ	0.05	Housing share
Capital producers		
χ	1.7	Investment adjustment cost
Retailers		
ϵ	6	Elasticity of substitution between retail goods
θ	0.75	Probability of not adjusting price
Banking sector		
m_h	0.8	Loan-to-value ratio for housing
m_k	0.5	Loan-to-value ratio for physical capital
Central bank		
ρ_r	0.8	Interest rate smoothing
κ_π	1.5	Feedback coefficient on inflation
κ_y	0.03	Feedback coefficient on output

Table 2: Steady state values of variables given the calibrated parameters in Table 1

	Perfect Banking Competition	Imperfect Banking Competition
Output y	0.801151	0.637727
Consumption c	0.572789	0.496773
Investment i	0.127018	0.081448
Physical Capital k	5.0807	3.25792
Real Price of Capital q	1	1
Bank Loan b	5.04189	2.30064
Labor l	0.332613	0.313858
Real Wage w	1.24447	1.04981
Gross Inflation Rate π	1	1
Gross Real Deposit Rate R_r	1.00503	1.00503
Gross Real Loan Rate R_{rb}	1.00503	1.02651
Real Price of Housing q_h	14.6938	10.9022
Entrepreneur's Housing h^E	0.220364	0.0886748
Leverage Ratio $\frac{b}{q_h h^E + qk}$	0.606093	0.544574
Ratio $\frac{qk}{b}$	2.06159	2.10566
Ratio $\frac{q_h h^E}{b}$	1.25628	1.28314
Lagrange Multiplier λ_2^E	0.247924	0.071994

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